THREE NUCLEON CALCULATIONS WITH LOCAL TENSOR FORCES

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Received 23 September 1969

Within the framework of the Faddeev equations, the triton binding energy, Coulomb energy, S'- and D-state probabilities are determined. The two-particle interactions thereby employed are of the local Yukawa type and include also tensor forces.

Recently a computational method was presented which enabled us to calculate in a simple way the binding energy and the wave function of the ground state of a three particle system and which is applicable even in the case of local interactions [1]. This method was applied to the triton problem where central local two-particle interactions for the triplet and singlet channels were used as input forces. A reasonable agreement was thereby obtained with experiment. It is of some interest to extend these calculations to include also tensor forces. In this note we present some of the results we have obtained in doing this.

The two-body potentials which have been used in these calculations are of the following type. In the first place, we have chosen the potential shape for the triplet channel to be given in the momentum representation by

$$V_t(\boldsymbol{p}|\boldsymbol{p}') = -\Lambda\left[\frac{1}{k^2+\alpha_t^2} + \gamma S_{12}\frac{k^2}{(k^2+\beta_t^2)^2}\right] \qquad (1)$$

where: $\boldsymbol{k} = \boldsymbol{p} - \boldsymbol{p}'$, $\boldsymbol{p} = \frac{1}{2\sqrt{m}}(\boldsymbol{k}_1 - \boldsymbol{k}_2)$,

$$S_{12} = \frac{3(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{k})(\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{k})}{k^2} - \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$$

and $\boldsymbol{k}$, i = 1,2 are the momenta of the particles. The parameters $\Lambda, \gamma, \alpha_t$ and β_t were determined from the triplet low energy data, i.e., the scattering length, deuteron binding energy, quadrupole moment and the D-state probability.

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Since the latter quantity is experimentally not very well known, we have used two different values for it (2.9 and 4.2%) in order to see what the effect of such a variation is in the three body problem. Secondly, the singlet potential used contains a soft-core repulsion and has the following form

$$V_s(\boldsymbol{p}|\boldsymbol{p}') = \frac{-\lambda_A}{k^2+\mu_A^2} + \frac{\lambda_R}{k^2+\mu_R^2} \qquad (2)$$

The parameters in eq. (2) were adjusted to the singlet scattering length and effective range. Moreover, the adjustment was made in such a way that good agreement with the experimental $^1S_0$ phase shifts was obtained up to 300 MeV lab energy. In order to study the sensitivity of the three-body calculations with respect to the singlet effective range we have used two different values for it (2.6 and 2.8 fm). The values found for the potential parameters together with the corresponding low energy two-body data are collected in table 1.

Having determined the above set of two-body interactions, we reduced the Faddeev equations by making a partial wave decomposition similarly as has been done in ref. 2 for the case of spinless particles. Keeping thereby only the s-wave part of the singlet channel and the s- and d-wave parts of the triplet channel, a coupled set of three integral equations in two continuous variables is derived from the Faddeev equations. In the derivation it is also assumed that the orbital angular momentum of the third particle, with respect to the other two particles, is in an l = 0 state.

Table 1
Theoretical and experimental two-nucleon low-energy parameters.

Singlet

λ_A (MeV)$^{1/2}$	μ_A (MeV)$^{1/2}$	λ_R (MeV)$^{1/2}$	μ_R (MeV)$^{1/2}$	a_s (fm)	r_s (fm)
5.08	11	12.00	20	-23.4	2.6
4.10	10	11.48	20	-23.4	2.8
Exp. (ref. 4)				-23.7	2.7

Triplet

Λ (MeV)$^{1/2}$	α_t (MeV)$^{1/2}$	γ	β_t (MeV)$^{1/2}$	a_t (fm)	E_D (MeV)	Q (fm^2)	P_D (%)
0.4647	5	-0.74	5	5.45	2.225	0.281	2.9
0.2232	3.5	-3.1	7	5.45	2.225	0.281	4.2
Exp. (ref. 4)				5.42	2.225	0.282	4-7

The resulting set of integral equations has been studied numerically in the same way as described in ref. 1. The results are shown in table 2. From this table, together with the results from ref. 1, it is clear that the inclusion of tensor forces reduces the binding energy (B.E.) of the triton considerably by 1 to 1.5 MeV depending very sensitively on the D-percentage. It is also interesting to note that the relative differences in B.E. between the two sets of D-percentage and singlet effective range are of the same order as indicated by the results obtained by Phillips [3] for the case of separable interactions. On the other hand, if we compare the absolute magnitude of B.E. of, for instance, 7.1 MeV, with calculations using various separable potentials ‡, we see that our local potentials give underbinding with respect to the experimental value while separable ones give overbinding due to the fact that the inclusion of a soft core repulsion and a tensor force in the local interaction decreases enormously the binding energy of the triton (from 12.1 MeV to 7.1 MeV) while the effect of similar separable potentials is much less pronounced. It is also amusing to note that our calculations with the set of potentials (1) and (2) are consistent with the latest variational results of Delves et al. [6] who obtained for the B.E. of triton 6.7 ± 0.7 MeV using the Hamada-Johnston potential and who ascribed the difference of 1.8 MeV with experiment to relativistic effects, the presence of three-body forces or to the assumed form of the potential. If one would neglect the first two possibilities, this would imply that quite low values of D-mixing and singlet effective range should be taken in our calculation. Of course, it should be said that in our calculations we have also neglected the contributions coming from other waves than s- and d-waves. As pointed out by several authors [6] for the simple cases studied by them, neglecting the higher partial waves is in fact a reasonable approximation. In order to get some feeling in our case for this, we have truncated the Faddeev equations into an effective two-channel problem by leaving out the d-wave contributions. Comparing the results with those obtained by solving the full three-channel problem, we find that there is a difference of only about 2% in binding energy and Coulomb energy. From this we expect that the contributions from the higher partial waves should indeed be quite negligible.

Concerning the Coulomb energy, calculated as in ref. 1, we see from table 2 that tensor forces do not improve the situation (although

‡ References can be found in refs. 4.

Table 2
Triton binding energy $B(^3H)$, Coulomb energy E_c, D-state probability $P(D)$ and S'-state probability $P(S')$ for different sets of potentials corresponding to a change in singlet effective range r_s and triplet D-state probability P_D. The numbers given in the last two columns are those obtained for the binding energy $B_t(^3H)$ and Coulomb energy E_c^t by restricting the Faddeev kernel to s-waves only.

r_s (fm)	P_D (%)	$B(^3H)$ (MeV)	E_c (MeV)	$P(D)$ (%)	$P(S')$ (%)	$B_t(^3H)$ (MeV)	E_c^t (MeV)
2.6	2.9	-8.1	0.73	3.5	1.6	-8.3	0.74
2.6	4.2	-7.5	0.70	5.7	1.3	-7.7	0.72
2.8	2.9	-7.5	0.71	3.5	1.9	-7.7	0.72
2.8	4.2	-7.1	0.68	5.7	1.5	-7.3	0.70

their effect is to increase the Coulomb energy), so that taking also into account a possible 5% effect due to the finite proton size, the values of E_c thus obtained is definitely smaller than the experimental value of 0.764 MeV, which would imply a charge asymmetry of nuclear forces as already suggested by Okamoto et al. [7].

Finally, we also calculated the D-state and S'-state probabilities in the triton. The S'-state probabilities contain a large numerical error of about 20% but give the correct magnitudes while the decrease with decreasing singlet effective range is in agreement with other results [8]. The D-state probabilities vary from 5.7% to 3.5% which is in agreement with numbers quoted by Delves [6] and Phillips [9] concerning their relative difference and agree with a 5 to 6% D-mixing in the triton quoted by Phillips for a triplet potential which gives 4% D-percentage.

This investigation is part of the research program of the "Stichting voor Fundamenteel Onderzoek der Materie" and was made possible by financial support from the "Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek". We are greatly indebted for the computational facilities offered by the "Stichting voor Fundamenteel Onderzoek der Materie" at the Zeeman Laboratory in Amsterdam.

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