Two Dimensional Estimation of Stochastic Volatility in The Hull-White Model Using Nonlinear Filtering

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Outline of Presentation

Motivation of research:

Application to pricing any contingent claims, risk management, etc.

- Mathematical model of the Hull-White Model in 2D
- Associated nonlinear filtering problems
- Maximum likelihood estimation of parameters
- Monte Carlo simulations
- Conclusions and remarks

The Hull-White Stochastic Volatility Model in 2D

Mathematical modelling

* Let Z_j and V_j be the log-return and volatility of stock j, $j = 1, 2$:

$$dZ_1(t) = (\mu_1 - \frac{1}{2}V_1(t))dt + \sqrt{V_1(t)}dB_t^1,\ Z_1(0) = 0$$

$$dZ_2(t) = (\mu_2 - \frac{1}{2}V_2(t))dt + \sqrt{V_2(t)}dB_t^2,\ Z_2(0) = 0$$

$$dV_1(t) = \beta_1 V_1(t)dt + \kappa_1 V_1(t)dB_t^3,\ V_1(0) = V_{10}$$

$$dV_2(t) = \beta_2 V_2(t)dt + \kappa_2 V_2(t)dB_t^4,\ V_2(0) = V_{20}$$

* Th BMs B^j defined on $(\Omega, \mathcal{F}, (\mathcal{F})_{t>0}, \mathbb{P})$ has the correlation structure:

$$dB_t^1 dB_t^2 = \rho_{Z_1 Z_2} dt,\ dB_t^1 dB_t^3 = \rho_{Z_1 V_1} dt,\ dB_t^2 dB_t^4 = \rho_{Z_2 V_2} dt$$

* Our objective: To determine $\mathbb{E}\{Z|\mathcal{F}_{t_i}\}$, and particularly $\mathbb{E}\{V|\mathcal{F}_{t_i}\}$ for all $t_1 < \ldots < t_i < \ldots < t_N$, where the information flow is given by $\mathcal{F}_t = \sigma\big((Z_s^1, Z_s^2), s \leq t\big)$

The Stochastic State Space Formulation

- The stochastic state space model

$$\begin{pmatrix} dZ_1(t) \\ dZ_2(t) \\ dV_1(t) \\ dV_2(t) \end{pmatrix} = \begin{pmatrix} \mu_1 - \frac{1}{2}V_1(t) \\ \mu_2 - \frac{1}{2}V_2(t) \\ \beta_1 V_1(t) \\ \beta_2 V_2(t) \end{pmatrix} dt + \begin{pmatrix} \sqrt{V_1(t)} & 0 & 0 & 0 \\ 0 & \sqrt{V_2(t)} & 0 & 0 \\ 0 & 0 & \kappa_1 V_1(t) & 0 \\ 0 & 0 & 0 & \kappa_2 V_2(t) \end{pmatrix} \begin{pmatrix} dB_t^1 \\ dB_t^2 \\ dB_t^3 \\ dB_t^4 \end{pmatrix}$$

- and the observation equation is given by

$$\mathbf{Y}_{t_i} = \begin{pmatrix} Z_1(t_i) \\ Z_2(t_i) \end{pmatrix} + \begin{pmatrix} e_1(t_i) \\ e_2(t_i) \end{pmatrix} \; ; \; \begin{pmatrix} e_1(t_i) \\ e_2(t_i) \end{pmatrix} \tilde{} \mathcal{N}\left(\mathbf{0}, \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix}\right)$$

- We assume that:
  - The log-returns $\mathbf{Z}$ is assumed to be observable
  - The BMs $\mathbf{B}$ has covariance matrix $\mathbf{Q}$ and is independent of Gaussian corrupted noises $\mathbf{e}$

Nonlinear Filtering Problems

A general state space model

- Assume that a general model for the state variables $\mathbf{x}(t) \in \mathbb{R}^n$ is given by

$$d\mathbf{x}(t) = \mathbf{f}[\mathbf{x}(t);\theta]dt + \mathbf{G}[\mathbf{x}(t);\theta]d\mathbf{B}(t), \mathbf{x}(t_0) = \mathbf{x}_0 \tag{1}$$

- $\mathbf{x}_0$ is a stochastic initial condition satisfying $\mathbb{E}(|\mathbf{x}_0|^2) < \infty$.
- The function $\mathbf{f} : \mathbb{R}^n \times \mathbf{R}^p \longmapsto \mathbb{R}^n$ and $\mathbf{G} : \mathbb{R}^n \times \mathbb{R}^p \longmapsto \mathbb{R}^{n\times d}$ are assumed to satisfy some regularilty conditions: growth and Lipschitz continuity condition to ensure the existence of a unique solution to (1)
- They are twice continuously differentiable with respect to $\mathbf{x}$
- The TPD of $\mathbf{x}(t)$, $f_{\mathbf{x}}(\xi,t \mid \rho,t')$ is defined as follows

$$f_{\mathbf{x}(t)|\mathbf{x}(t')}(\xi|\,\rho) \triangleq f_{\mathbf{x}}(\xi,t|\,\mathbf{x}(t') = \rho) \triangleq f_{\mathbf{x}}(\xi,t|\,\rho,t') \tag{2}$$

Fokker-Planck partial differential equations (PDE): Continued

* TPD solves Fokker-Planck PDE

$$\frac{\partial f_{\mathbf{x}}\left(\xi,t \middle| \rho,t'\right)}{\partial t} = -\sum_{i=1}^{n} \frac{\partial}{\partial \xi_i}\{f_{\mathbf{x}}\left(\xi,t \middle| \rho,t'\right) f_i[\xi,t]\}$$
$$+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2}{\partial \xi_i \partial \xi_j}\left[f_{\mathbf{x}}\left(\xi,t \middle| \rho,t'\right)\{\mathbf{G}[\xi,t]\mathbf{Q}(t)\mathbf{G}^T[\xi,t]\}_{ij}\right]$$

* The appropriate initial condition of the PDE is simply

$$f_{\mathbf{x}}\left(\xi,t \middle| \rho,t'\right) = \delta(\xi-\rho)$$

* We assume that the derivatives

$$\frac{\partial f_{\mathbf{x}}}{\partial t},\ \frac{\partial}{\partial \xi_i}\{f_{\mathbf{x}} f_i\},\ \frac{\partial^2}{\partial \xi_i \partial \xi_j}\left[f_{\mathbf{x}}\{\mathbf{G}\mathbf{Q}\mathbf{G}^T\}_{ij}\right]$$

exist and are continuous for all i and j.

Mean and covariance propagation equations: Continued

* Without knowledge for all t of the entire density $f_{\mathbf{x}}\left(\xi, t \middle| \rho, t'\right)$ or $f_{\mathbf{x}(t)}(\xi)$, we might try to achieve a partial description of the density by propagating a finite number of its moments:

$$\begin{aligned}\mathbf{m}_x(t) &= \int_{-\infty}^{\infty} \xi f_{\mathbf{x}(t)}(\xi) d\xi \\ \mathbf{P}_x(t) &= \int_{-\infty}^{\infty} \xi \xi^T f_{\mathbf{x}(t)}(\xi) d\xi - \mathbf{m}_x(t)\mathbf{m}_x^T(t)\end{aligned}$$

* Using Fokker-Planck PDE applied to the above eqn., we obtain:

$$\dot{\mathbf{m}}_x(t) = \mathbb{E}\{\mathbf{f}[\mathbf{x}(t);\theta]\} \tag{3}$$

$$\begin{aligned}\dot{\mathbf{P}}_x(t) &= \left[\mathbb{E}\{\mathbf{f}[\mathbf{x}(t);\theta]\mathbf{x}^T(t)\} - \mathbb{E}\{\mathbf{f}[\mathbf{x}(t);\theta]\mathbf{m}_x^T(t)\}\right] \\ &\quad + \left[\mathbb{E}\{\mathbf{x}(t)\mathbf{f}^T[\mathbf{x}(t);\theta]\} - \mathbf{m}_x(t)\mathbb{E}\{\mathbf{f}^T[\mathbf{x}(t);\theta]\}\right] \\ &\quad + \mathbb{E}\{\mathbf{G}[\mathbf{x}(t);\theta]\mathbf{Q}(t)\mathbf{G}^T[\mathbf{x}(t);\theta]\}\end{aligned} \tag{4}$$

An approximate filter : Continued

* It can be shown that

$$f_{\mathbf{x}(t)|\mathbf{Y}(t_{i-1})}\left(\xi|\,\mathcal{F}_{i-1}\right)$$

solves the Fokker-Planck PDE on the interval $[t_{i-1}, t_i)$, with initial condition

$$f_{\mathbf{x}(t_{i-1})|\mathbf{Y}(t_{i-1})}\left(\xi|\,\mathcal{F}_{i-1}\right)$$

* Consider the following time propagation of the conditional mean and variance of the state $\mathbf{x}(t)$:

$$\begin{aligned}\hat{\mathbf{x}}_{t|t_{i-1}} &= \mathbb{E}[\mathbf{x}(t)|\,\mathcal{F}_{t_{i-1}}] \\ \mathbf{P}_{t|t_{i-1}} &= \mathbb{E}\{[\mathbf{x}(t)-\mathbf{x}_{t|t_{i-1}}][\mathbf{x}(t)-\mathbf{x}_{t|t_{i-1}}]^T\Big|\,\mathcal{F}_{t_{i-1}}]\end{aligned}$$

* Next define $\widehat{(\cdot)} = \mathbb{E}\{(\cdot)|\,\mathbf{Y}(t_{i-1}) = \mathbf{Y}_{i-1}\}$. Following eqns. (3)-(4) on $\mathbf{Y}_{i-1}$:

$$\frac{d\hat{\mathbf{x}}_{t|t_{i-1}}}{dt} = \mathbf{f}[\widehat{\mathbf{x}(t);\theta}] \tag{5}$$

$$\begin{aligned}\frac{d\mathbf{P}_{t|t_{i-1}}}{dt} &= \{\widehat{\mathbf{f}[\mathbf{x}(t);\theta]\mathbf{x}^T}(t) - \mathbf{f}[\widehat{\mathbf{x}(t);\theta}]\mathbf{x}^T_{t|t_{i-1}}\} \\ &+\{\mathbf{x}(t)\widehat{\mathbf{f}^T[\mathbf{x}(t)};\theta] - \mathbf{x}_{t|t_{i-1}}\mathbf{f}^T[\widehat{\mathbf{x}(t);}\theta]\} \\ &+\mathbf{G}[\mathbf{x}(t);\theta]\widehat{\mathbf{Q}(t)}\mathbf{G}^T[\mathbf{x}(t);\theta]\end{aligned} \tag{6}$$

* By Taylor expansions of $\mathbf{f}[\mathbf{x}(t);\theta]$ and $\mathbf{G}[\mathbf{x}(t);\theta]$ around $\hat{\mathbf{x}}_{t|t_i}$, truncating after the second order terms and taking expectations

$$\frac{d\hat{\mathbf{x}}_{t|t_i}}{dt} = \mathbf{f}[\widehat{\mathbf{x}}_{t|t_i};\theta] + \hat{\mathbf{b}}_{p\,t|t_i} \tag{7}$$

$$\begin{aligned}\frac{d\mathbf{P}_{t|t_i}}{dt} &= \mathbf{F}[\widehat{\mathbf{x}}_{t|t_i};\theta]\mathbf{P}_{t|t_i} + \mathbf{P}_{t|t_i}\mathbf{F}^T[\widehat{\mathbf{x}}_{t|t_i};\theta] \\ &+\mathbf{G}[\mathbf{x}(t);\theta]\widehat{\mathbf{Q}(t)\mathbf{G}}^T[\mathbf{x}(t);\theta]\end{aligned} \tag{8}$$

* where $\mathbf{F}[\widehat{\mathbf{x}}_{t|t_i};\theta]$ is the n-by-n partial derivative matrix

$$\mathbf{F}[\widehat{\mathbf{x}}_{t|t_i};\theta] \triangleq \left.\frac{\partial \mathbf{f}[\mathbf{x}(t);\theta]}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}_{t|t_i}}$$

* Next, let us define

$$\tilde{\mathbf{G}}[\mathbf{x}(t);\theta] \triangleq \mathbf{G}[\mathbf{x}(t);\theta]\mathbf{Q}^{1/2}(t).$$

The last term in (7) and (8) is an n-by-n matrix with ij element defined by

$$\begin{aligned}[\widehat{\mathbf{GQG}^T}]_{ij} = \sum_{k=1}^{d}\Bigg[&\tilde{G}_{ik}\tilde{G}_{kj}^T + tr\left\{\left(\frac{\partial \tilde{G}_{ik}}{\partial \mathbf{x}}^T \frac{\partial \tilde{G}_{kj}^T}{\partial \mathbf{x}}\right)\mathbf{P}\right\} + \frac{1}{2}\tilde{G}_{ik} tr\left\{\frac{\partial^2 \tilde{G}_{kj}^T}{\partial \mathbf{x}^2}\mathbf{P}\right\} \\ &+ \frac{1}{2} tr\left\{\mathbf{P}\frac{\partial^2 \tilde{G}_{ik}}{\partial \mathbf{x}^2}\right\}\tilde{G}_{kj}^T + \frac{1}{4} tr\left\{\frac{\partial^2 \tilde{G}_{ik}}{\partial \mathbf{x}^2}\mathbf{P}\right\} tr\left\{\frac{\partial^2 \tilde{G}_{kj}^T}{\partial \mathbf{x}^2}\mathbf{P}\right\} \\ &+ \frac{1}{2} tr\left\{\frac{\partial^2 \tilde{G}_{ik}}{\partial \mathbf{x}^2}\mathbf{P}\frac{\partial^2 \tilde{G}_{kj}^T}{\partial \mathbf{x}^2}\mathbf{P}\right\}\Bigg]\end{aligned}$$

* The bias correction terms $\widehat{\mathbf{b}}_{p\,t|t_i}$ is the n-vector with kth component given by

$$\widehat{b}_{p\,t|t_i}^{\,k} \triangleq \frac{1}{2} tr \left\{ \frac{\partial^2 f_k[\widehat{\mathbf{x}}_{t|t_i};\theta]}{\partial \mathbf{x}^2} \mathbf{P}_{t|t_i} \right\} \tag{9}$$

* The next job is how to determine initial conditions to be able to run the ODE (7) and (8).

Optimal State Estimator: Projection Theorem

* Let $\mathcal{L}_2$ be an Hilbert Space of all random variables X for which

$$\mathbb{E}\{|X|^2\} < \infty$$

endowed with the inner product:

$$< X, Y > = \mathbb{E}XY; \quad X, Y \in \mathcal{L}_2$$

* Two elements in $\mathcal{L}_2$ are ortogonal if $\mathbb{E}XY = 0$.
* We denote the norm in $\mathcal{L}_2$ by $\|\cdot\|$ s.t. $\|X\|^2 \triangleq < X, X >, X \in \mathcal{L}_2$. It can be shown that $\mathcal{L}_2$ is complete
* Projection Theorem: If $\mathcal{V}$ is any complete subspace of $\mathcal{L}_2$, any element $X \in \mathcal{L}_2$ can be projected onto $\mathcal{V}$ uniquely in the sense:

$$\exists \text{ a unique } V^* \in \mathcal{V} \text{ s.t. } < X - V^*, V > = 0 \quad \forall V \in \mathcal{V}$$

* Let $\mathcal{L}_2 \ni \mathbf{Y} = (Y_1, \cdots, Y_n)$. We want to define $\mathbb{E}\{X|\mathbf{Y}\}$, $X \in \mathcal{L}_2$. Let $g : \mathbb{R}^n \longrightarrow \mathbb{R}$ be any Borel measurable function. Consider

$$\mathcal{L}_{\mathbf{Y}} = \Big\{ Y_g = g(\mathbf{Y}), \quad \text{s.t.} \quad \mathbb{E}\{|Y_g|^2\} < \infty \Big\}$$

  Then $\mathcal{L}_{\mathbf{Y}}$ is also complete and $\mathcal{L}_{\mathbf{Y}} \subset \mathcal{L}_2$

* By Projection Theorem: $\exists Y_g^* \in \mathcal{L}_{\mathbf{Y}}$, uniquely, s.t. $< X - Y_g^*, Y_g >= 0$ for any $Y_g \in \mathcal{L}_{\mathbf{Y}}$. This induces that $\mathbb{E}\{X|\mathbf{Y}\} \equiv Y_g^*$. Thus $\mathbb{E}\{X|\mathbf{Y}\}$ is the unique projection of X onto $\mathcal{L}_{\mathbf{Y}}$ provided that $X, \mathbf{Y} \in \mathcal{L}_2$

* Next, define innovation process: $\nu_k \triangleq Y_k - \mathbb{E}\{Y_k|\mathbf{Y}_{k-1}\}$.

* Let Z and $\{Y_k\}$ be jointly Gaussian. It is not difficult to see that

$$\mathbb{E}\{Z|\mathbf{Y}_k\} = \mathbb{E}\{Z|\nu_k\}$$

* Knowing that $\nu_j \perp \nu_k$ for $j \neq k$, it can be shown that

$$\begin{aligned}\widehat{X}_{k+1|k+1} &= \mathbb{E}\{X_{k+1}|\mathbf{Y}_{k+1}\} = \mathbb{E}\{X_{k+1}|\nu_{k+1}\} \\ &= \mathbb{E}\{X_{k+1}\} + Cov\big(X_{k+1}, \nu_{k+1}\big)\{Var(\nu_{k+1})\}^{-1}\nu_{k+1} \\ &= \mathbb{E}\{X_{k+1}|\{\nu_k\}\} + Cov\big(X_{k+1}, \nu_{k+1}\big)\{Var(\nu_{k+1})\}^{-1}\nu_{k+1}\end{aligned} \tag{10}$$

* Now define the Kalman gain matrix K_{k+1} as follows

$$K_{k+1} = Cov\big(X_{k+1}, \nu_{k+1}\big)\{Var(\nu_{k+1})\}^{-1}$$

* Then, the equation (10) can be re-written as:

$$\begin{aligned}\widehat{X}_{k+1|k+1} &= \widehat{X}_{k+1|k} + K_{k+1}\nu_{k+1} \\ &= \widehat{X}_{k+1|k} + K_{k+1}\Big(Y_{k+1} - \mathbb{E}\{Y_{k+1}|\nu_k\}\Big)\end{aligned}$$

* The initial conditions for the equations (7) and (8), $\hat{\mathbf{x}}_{t_i|t_i}$ and $\mathbf{P}_{t_i|t_i}$, are provided

by the following observations update at time t_i

$$\begin{aligned}
\mathbf{A}(t_i) &= \mathbf{H}[\widehat{\mathbf{x}}_{t_i|t_{i-1}};\theta]\mathbf{P}_{t_i|t_{i-1}}\mathbf{H}^T[\widehat{\mathbf{x}}_{t_i|t_{i-1}};\theta] && (11)\\
&\quad +\hat{\mathbf{B}}_{ot_i|t_{i-1}} + \mathbf{\Sigma}(t_i) && (12)\\
\mathbf{K}(t_i) &= \mathbf{P}_{t_i|t_{i-1}}\mathbf{H}^T[\widehat{\mathbf{x}}_{t_i|t_{i-1}};\theta]\mathbf{A}^{-1}(t_i) && (13)\\
\widehat{\mathbf{x}}_{t_i|t_i} &= \widehat{\mathbf{x}}_{t_i|t_{i-1}} + \mathbf{K}(t_i)\{\mathbf{y}(t_i) - \mathbf{h}[\widehat{\mathbf{x}}_{t_i|t_{i-1}};\theta] - \widehat{\mathbf{b}}_{ot_i|t_{i-1}}\} && (14)\\
\mathbf{P}_{t_i|t_i} &= \mathbf{P}_{t_i|t_{i-1}} - \mathbf{K}(t_i)\mathbf{H}[\widehat{\mathbf{x}}_{t_i|t_{i-1}};\theta]\mathbf{P}_{t_i|t_{i-1}} && (15)
\end{aligned}$$

* where $\mathbf{H}[\widehat{\mathbf{x}}_{t_i|t_{i-1}};\theta]$ is defined as the m-by-n partial derivative matrix

$$\mathbf{H}[\widehat{\mathbf{x}}_{t_i|t_{i-1}};\theta] \triangleq \left.\frac{\partial \mathbf{h}[\mathbf{x}(t_i);\theta]}{\partial \mathbf{x}}\right|_{\mathbf{x}=\widehat{\mathbf{x}}_{t_i|t_{i-1}}} \qquad (16)$$

* and the bias correction term $\hat{\mathbf{b}}_{ot_i|t_{i-1}}$ is the m-vector with kth component given as

$$\widehat{b_o}^{k}_{t_i|t_{i-1}} = \frac{1}{2} tr \left\{ \frac{\partial^2 h_k[\widehat{\mathbf{x}}_{t_i|t_{i-1}}; \theta]}{\partial \mathbf{x}^2} \mathbf{P}_{t_i|t_{i-1}} \right\} \tag{17}$$

* and $\hat{\mathbf{B}}_{ot_i|t_{i-1}}$ is an m-by-m matrix with the kl element as

$$\hat{B}^{kl}_{ot_i|t_{i-1}} \triangleq \frac{1}{2} tr \left\{ \frac{\partial^2 h_k[\hat{\mathbf{x}}_{t_i|t_{i-1}}]}{\partial \mathbf{x}^2} \mathbf{P}_{t_i|t_{i-1}} \frac{\partial^2 h_l[\hat{\mathbf{x}}_{t_i|t_{i-1}}]}{\partial \mathbf{x}^2} \mathbf{P}_{t_i|t_{i-1}} \right\} \tag{18}$$

Systems Parameter Estimation

* The maximum likelihood method is based on an assumption of normality for the innovations given in the curly bracket in equation 14:

$$\epsilon_{t_i}(\theta) \equiv \mathbf{Y}_{t_i} - \widehat{\mathbf{Y}}_{t_i|t_{i-1}} = \mathbf{Y}_{t_i} - \mathbf{h}[\widehat{\mathbf{x}}_{t_i|t_{i-1}};\theta] - \widehat{\mathbf{b}}_{o t_i|t_{i-1}}$$

Given all the observations $\mathcal{Y}^N = [\mathbf{Y}_1, \ldots, \mathbf{Y}_N]$

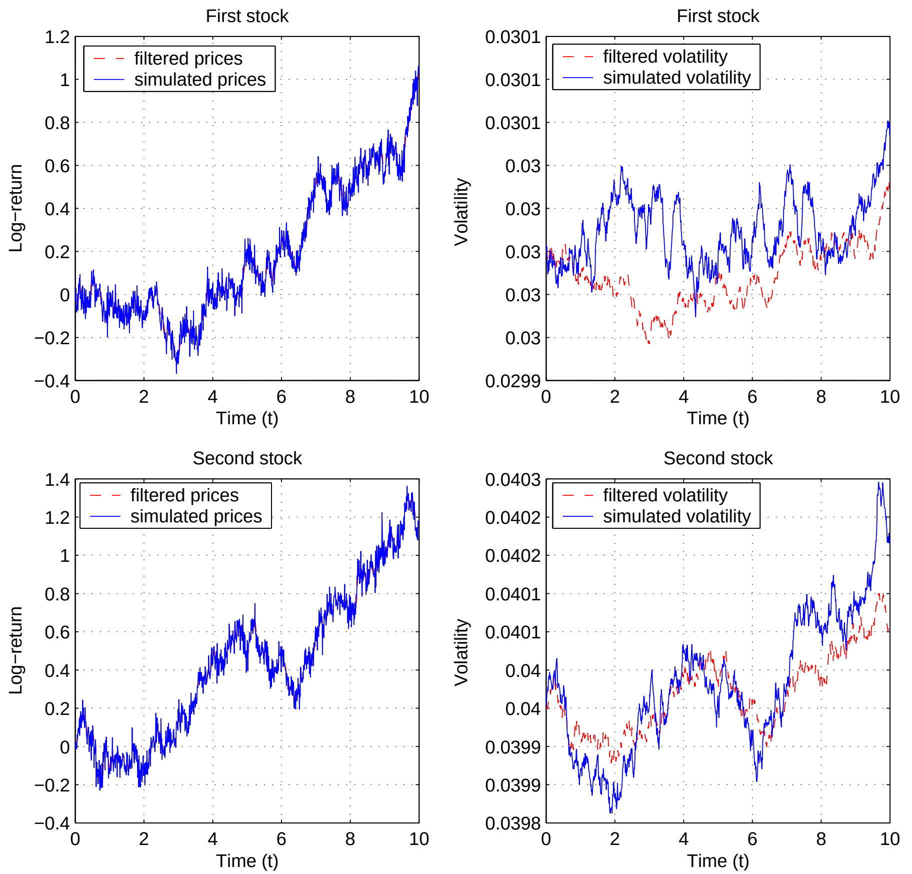
* By assuming that the innovations are normal with zero mean and covariance matrix as given in the equation (12), it is convenient to consider the logarithm of $\widetilde{\mathcal{L}}$:

$$l(\theta) = -\ln \widetilde{\mathcal{L}}(\theta;\mathcal{F}_N) = \frac{1}{2}\sum_{i=1}^{N}\left(\ln\det(\mathbf{A}_{t_i|t_{i-1}}) + \epsilon^T_{t_i}(\theta)\mathbf{A}^{-1}_{t_i|t_{i-1}}\epsilon_{t_i}(\theta) + m\log 2\pi\right) \tag{19}$$

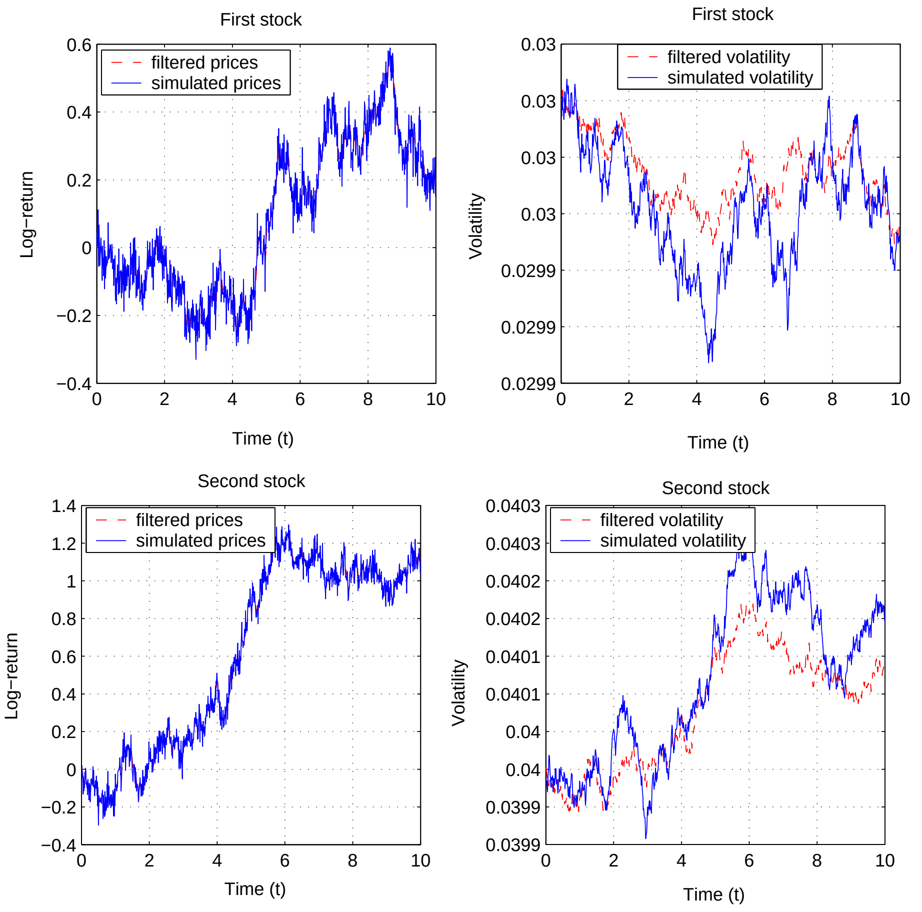
* such that the maximum likelihood estimate is determined by minimizing the negative log-likelihood function, i.e.,

$$\widehat{\theta} = \arg\min_{\theta\in\mathbf{\Theta}} l(\theta) \tag{20}$$

Simulation Studies for $\rho_{Z_1 Z_2} = 0$



Simulation Studies for $\rho_{Z_1 Z_2} = 0.2$



Concluding Remarks

- ⋆ Due to the nonlinearity nature of the problem, we have used an approximate Gaussian second order filtering equations to estimate stochastic volatility.
- ⋆ The proposed filtering equations have been shown to capture the dynamics of the simulated stochastic volatility reasonably good, particularly when we allow correlation between the two asset under consideration.
- ⋆ It has been common knowledge to model the innovation of the log-return of stock price by Lévy processes. By doing this, the model of stock price is expected to be close to reality as possible.
- ⋆ But, there is no guarantee that the current proposed filtering will be able to cope with the estimation problem since the underlying assumption on stock price is entirely different. We keep this task for possible future work.
- ⋆ THANK YOU!

Main References

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