Coherently scattering atoms from an excited Bose-Einstein condensate

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We consider scattering atoms from a fully Bose-Einstein condensed gas. If we take these atoms to be identical to those in the Bose-Einstein condensate, this scattering process is to a large extent analogous to Andreev reflection from the interface between a superconducting and a normal metal. We determine the scattering wave function in both the absence and the presence of a vortex. Our results show a qualitative difference between these two cases that can be understood as due to an Aharonov-Bohm effect. It leads to the possibility to experimentally study vortices in a nondestructive way.

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I. INTRODUCTION

Among the most striking features of superfluidity in a Bose-condensed fluid are the quantization of circulation, and the presence of persistent currents. Indeed, in the case of superfluid helium, vortices have been studied extensively [1–4]. Because of the strongly interacting nature of such a Bose-condensed liquid, the understanding of these phenomena is, however, mainly phenomenological. In contrast, in weakly interacting Bose-condensed gases the theoretical study of vortices can be based on a microscopic theory. As a result, the achievement of Bose-Einstein condensation in trapped alkali vapors has generated renewed theoretical as well as experimental interest in this subject.

Problems that have been studied in detail theoretically are the stability of a vortex in a harmonic trapping potential, the related question of its experimental creation, and the detection of vortices [5–18]. The creation of a stable vortex appears to be possible by rotating an external trapping potential that is sufficiently anisotropic with a frequency larger than some critical value. In this case, a vortex configuration becomes the thermodynamic equilibrium of the system. This equilibrium can be reached in two ways, either by first cooling the gas through the transition temperature and then rotating the trapping potential, or by first rotating and then cooling the gas. The latter seems to be the most favorable experimentally [18]. Indeed, this scheme has recently been carried out successfully [19]. In this experiment, the core of the vortex was detected by releasing the Bose-condensed cloud from the trap and allowing it to expand for some time. In the ballistic expansion following the release, the vortex core expands at least as fast as the transverse size of the condensate [11], and it becomes possible to detect the core by means of optical imaging. States were observed with up to four vortices present, and a lifetime in the range from 400 to 1000 ms.

A different method for creating vortices, which has been used successfully in a two-component Bose-condensed mixture, is phase imprinting [20–22]. Here, laser beams are used to impose a phase pattern upon the condensate, which then forces the condensate wave function into a vortex state. This method has also been used to create solitons in trapped Bose-condensed vapors [23].

Other theoretical proposals to study vortices include the interference fringes between two condensates containing vortices, which will exhibit dislocations [12,13], and the excitation frequencies of the collective modes of the condensate, which are shifted due to the presence of a vortex [8,14,15]. Finally, it has been proposed to study vortices by optical means [16].

In addition to these ideas, we suggest and investigate in this paper a nondestructive method that is based on the scattering of identical atoms off the Bose-condensed cloud, analogous to the occurrence of Andreev reflection [24] from a superconductor–normal-metal interface [25]. In the latter case, the scattered electrons probe both the magnitude and the phase of the superconducting order parameter. A similar experiment with a trapped Bose-condensate will, aside from the condensate density, also probe gradients in the phase of the Bose-condensed cloud. This is desirable, since the vortex core is typically small compared to the overall size of the condensate, whereas the superfluid velocity due to the presence of the vortex extends over the entire Bose-condensed cloud. Because the velocity field of a condensate with and without a vortex is qualitatively different, the scattering of atoms might be a possible way to detect a vortex. Moreover, this method is nondestructive, and allows for an *in situ* study of the time evolution of a vortex.

Note that a similar idea has been put forward recently in the context of superfluid ^{4}He. In particular, the proposal is to study the quantum sticking, scattering and transmission of a beam of ^{4}He atoms from superfluid ^{4}He slabs [26]. In an experiment with a dilute Bose condensate, the energy of the incoming atoms has to be rather small to see an effect, namely, of the order of the chemical potential of the condensate. A convenient way to realize this experimentally is by using optical means to extract the atoms that are to be scattered from the condensate itself. After displacing these atoms with respect to the condensed cloud, they are subsequently released and measured by the usual time-of-flight measurements or *in situ* optical imaging [27]. Note that the extracted atoms will be displaced over a large distance, such that they are spatially separated from the condensate. If one would displace the atoms only slightly, the experiment suggested by us would become equivalent to an experiment where the condensate is exited by a perturbation of the external trapping frequency. However, the two cases of small and large displacement are qualitatively different because in the former

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case there is only one dynamical system, whereas in the latter case a description of the scattering process requires the introduction of two dynamical systems interacting with each other, i.e., the incoming cloud of atoms and the condensate. The incoming cloud of atoms can, for example, collide with a condensate in its ground state, leaving the condensate in an excited state after the atoms are again spatially separated from the condensate. The treatment of the incoming, noncondensed atoms as a separate dynamical system is also used in the study of hydrodynamics [28], collective modes [29,30], and in kinetic theories describing the nonequilibrium dynamics of Bose-Einstein condensates [31,32]. In most of these works, the complete dynamics of the condensed and noncondensed part of the gas is described by a Boltzmann equation coupled to an appropriate nonlinear Schrödinger equation. Moreover, in the collisionless limit, the Boltzmann equation is equivalent to the long-wavelength limit of the Bogoliubov–de Gennes equations.

The necessity for two dynamical systems to describe the experiment proposed here becomes even more clear when we consider a more clean version of it, where a beam of atoms enters the region in space where the condensate is trapped, collides with the condensate, and then leaves this region again. In this case, there are well-defined in and out states, corresponding to product states of the condensate wave function, which may be excited, and the noncondensed atoms. During the collision, the atoms interact with the condensate through the usual normal and anomalous mean fields, but the condensate still has a dynamics of its own described by the nonlinear Schrödinger equation. After the collision, the condensate may be in a different state than before the collision, corresponding to different possible scattering channels.

We have organized the paper as follows. In Sec. II, we first discuss in some detail the theory used to describe the scattering process. In Sec. III, we then present and discuss our numerical results. In Sec. IV, we end with some conclusions.

II. THEORY

Bearing the previous remarks in mind, we consider the whole scattering process to take place in an external confining potential. The scattering of an identical particle off the condensate can be described by the time-dependent Bogoliubov–de Gennes equations. To lowest-order approximation, the condensate density profile can be taken to be static, which implies that the time dependence of the mean-field interaction due to the possible excitation of one or more collective modes is neglected. In order to be as general as possible, we will show below how to extend this treatment to take into account the different scattering channels corresponding to a possible excitation of the condensate. In the static approximation, however, the coherence factors $f(\mathbf{r},t)$ and $g(\mathbf{r},t)$ for the atom satisfy

$$i\hbar\frac{\partial}{\partial t}\begin{bmatrix} f(\mathbf{r},t) \\ g(\mathbf{r},t)\end{bmatrix}=\begin{bmatrix} H & -V \\ V^* & -H^*\end{bmatrix}\begin{bmatrix} f(\mathbf{r},t) \\ g(\mathbf{r},t)\end{bmatrix}, \tag{1}$$

where

$$H=-\frac{\hbar^2\nabla^2}{2m}+V_{\mathrm{ext}}(\mathbf{r})-\mu+2T^{2\mathrm{B}}\rho(\mathbf{r}) \tag{2a}$$

and

$$V=T^{2\mathrm{B}}\rho(\mathbf{r})\exp[2i\chi(\mathbf{r})]. \tag{2b}$$

Here, $T^{2\mathrm{B}}=4\pi\hbar^2 a/m$ is the s-wave approximation for the two-body scattering matrix and $V_{\mathrm{ext}}(\mathbf{r})=\Sigma_i m\omega_i^2 r_i^2/2$ denotes the external trapping potential. The interatomic scattering length is given by a. Furthermore, the static density profile of the condensate is denoted by $\rho(\mathbf{r})$ and its phase by $\chi(\mathbf{r})$. Outside the condensate, $g(\mathbf{r},t)$ has to be zero and Eq. (1) reduces to the Schrödinger equation for an atom in a harmonic potential.

A. Dynamical condensate

Of course, the colliding atom can, in principle, excite the condensate. To include the possibility of exciting collective modes, we simply have to replace $\rho(\mathbf{r})$ and $\chi(\mathbf{r})$ by their operator counterparts, $\hat{\rho}(\mathbf{r},t)$ and $\hat{\chi}(\mathbf{r},t)$, respectively. The latter two can be expanded in terms of the creation and annihilation operators $\hat{b}_i^\dagger$ and $\hat{b}_i$ of the collective modes with mode functions $u_i(\mathbf{r},t)=u_i(\mathbf{r})\exp(-i\omega_i t)$ and $v_i(\mathbf{r},t)=v_i(\mathbf{r})\exp(-i\omega_i t)$ as

$$\hat{\rho}(\mathbf{r},t)=\rho(\mathbf{r})+\sqrt{\rho(\mathbf{r})}\sum_i \{[u_i(\mathbf{r})-v_i(\mathbf{r})]\hat{b}_i e^{-i\omega_i t}+[u_i^*(\mathbf{r})-v_i^*(\mathbf{r})]\hat{b}_i^\dagger e^{i\omega_i t}\} \tag{3a}$$

and

$$\hat{\chi}(\mathbf{r},t)=\chi(\mathbf{r})+\frac{i}{2\sqrt{\rho(\mathbf{r})}}\sum_i \{[u_i(\mathbf{r})+v_i(\mathbf{r})]\hat{b}_i e^{-i\omega_i t}-[u_i^*(\mathbf{r})+v_i^*(\mathbf{r})]\hat{b}_i^\dagger e^{i\omega_i t}\}, \tag{3b}$$

where $\hbar\omega_i$ are the energy quanta of the collective modes. The mode functions satisfy the orthonormality condition $\int d\mathbf{r}[u_i^*(\mathbf{r})u_j(\mathbf{r})-v_i^*(\mathbf{r})v_j(\mathbf{r})]=\delta_{ij}$ and are usually defined by [33]

$$\psi(\mathbf{r},t)=e^{-i\mu t/\hbar}e^{i\chi(\mathbf{r})}\left\{\sqrt{\rho(\mathbf{r})}+\sum_i [u_i(\mathbf{r})e^{-i\omega_i t}-v_i^*(\mathbf{r})e^{i\omega_i t}]\right\}, \tag{4}$$

where $\psi(\mathbf{r},t)$ is the condensate wave function that satisfies the Gross-Pitaevskii equation. The Bogoliubov–de Gennes equations that follow from inserting Eq. (4) into the Gross-Pitaevskii equation and linearizing around equilibrium are again of the form of Eq. (1), but now with $H=-\hbar^2[\nabla+i\nabla\chi]^2/2m+V_{\mathrm{ext}}(\mathbf{r})-\mu+2T^{2\mathrm{B}}\rho(\mathbf{r})$ and $V=T^{2\mathrm{B}}\rho(\mathbf{r})$. At this point, we emphasize that although $\{f(\mathbf{r},t),g(\mathbf{r},t)\}$ and $\{u(\mathbf{r},t),v(\mathbf{r},t)\}$ satisfy similar equations, they do not describe the same physical quantities. The wave functions

$f(\mathbf{r},t)$ and $g(\mathbf{r},t)$ describe a noncondensed particle scattering off a condensate, whereas the mode functions $u(\mathbf{r},t)$ and $v(\mathbf{r},t)$ give to the dynamics of the condensate as governed by the Gross-Pitaevskii equation. As mentioned before, this is similar in spirit to a treatment of the dynamical properties of a partially Bose-condensed gas by means of a Gross-Pitaevskii equation for the condensate coupled to a Boltzmann equation for the noncondensed particles.

If we denote by $|\alpha\rangle$ the number states in the Fock-space spanned by the above creation and annihilation operators, the different channels of our scattering problem are given by

$$\begin{bmatrix} f^{\alpha}(\mathbf{r},t) \\ g^{\alpha}(\mathbf{r},t) \end{bmatrix} \otimes |\alpha\rangle, \tag{5}$$

and the equations of motion for these different channels read

$$i\hbar \frac{\partial}{\partial t}\begin{bmatrix} f^{\alpha}(\mathbf{r},t) \\ g^{\alpha}(\mathbf{r},t) \end{bmatrix} = \sum_{\alpha'} \langle \alpha | \begin{bmatrix} \hat{H} & -\hat{V} \\ \hat{V}^{\dagger} & -\hat{H}^{\dagger} \end{bmatrix} | \alpha' \rangle \begin{bmatrix} f^{\alpha'}(\mathbf{r},t) \\ g^{\alpha'}(\mathbf{r},t) \end{bmatrix}, \tag{6}$$

where

$$\hat{H} = -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{ext}}(\mathbf{r}) - \mu + 2T^{2\text{B}}\hat{\rho}(\mathbf{r},t) \tag{7a}$$

and

$$\hat{V} = T^{2\text{B}}\hat{\rho}(\mathbf{r},t)\exp[2i\hat{\chi}(\mathbf{r},t)]. \tag{7b}$$

Again, outside the condensate, $g^{\alpha}(\mathbf{r},t)$ has to be zero. It is clear that if the condensate is initially in the ground state $|0\rangle$, after colliding with the incoming atom it can be in the ground state or any of the excited states. Note that $|0\rangle$ can in principle be an arbitrary solution of the Gross-Pitaevskii equation and can in particular also be a vortex or a kink solution.

B. Static condensate

In the remainder of this work, we apply the static approximation and include only the ground state $|0\rangle$. The resulting equations describing our scattering problem then reduce to Eq. (1). This is expected to be accurate in the case of the scattering of a single atom at energies several times larger than the chemical potential. At these energies, the kinetic energy of an incoming particle is still relatively large and the overlap of its wave function with that of the low-lying collective modes is small. In addition, we take the external potential to be rotationally symmetric around the z axis, i.e., $\omega_x = \omega_y \equiv \omega_r$, and approximate the cigar-shaped traps used in experiments by assuming translational invariance in the z direction. It is then convenient to write the Laplace operator in cylindrical coordinates (r,z,ϕ), i.e., $\nabla^2 = (1/r)(\partial/\partial r) + (\partial^2/\partial r^2) + (1/r^2)(\partial^2/\partial \phi^2) + (\partial^2/\partial z^2)$ and expand $f(\mathbf{r},t)$ and $g(\mathbf{r},t)$ as

$$f(\mathbf{r},t) = \sum_n \frac{f_n(r,t)}{\sqrt{r}} e^{in\phi} \tag{8a}$$

and

$$g(\mathbf{r},t) = \sum_n \frac{g_n(r,t)}{\sqrt{r}} e^{in\phi}. \tag{8b}$$

Inserting this expansion into Eq. (1), the Bogoliubov–de Gennes equations for the wave functions $f_n(r,t)$ and $g_n(r,t)$ become

$$i\hbar \frac{\partial}{\partial t}\begin{bmatrix} f_n(r,t) \\ g_n(r,t) \end{bmatrix} = \sum_{n'} \begin{bmatrix} H_{nn'} & V_{nn'} \\ -V_{n'n} & -H_{nn'} \end{bmatrix} \begin{bmatrix} f_{n'}(r,t) \\ g_{n'}(r,t) \end{bmatrix}. \tag{9}$$

Here, the matrix elements $H_{nn'}$ and $V_{nn'}$ equal

$$H_{nn'} = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + V_{\text{ext}}(r) - \mu + \frac{\hbar^2(n^2 - 1/4)}{2mr^2} + 2T^{2\text{B}}\rho(r) \right]\delta_{n,n'} \tag{10a}$$

and

$$V_{nn'} = T^{2\text{B}}\rho(r)\,\delta_{n,n'+2l}. \tag{10b}$$

Note that we consider the solution of Eq. (1) in the presence of a vortex with a winding number l that is aligned in the direction of the z axis. This implies that the condensate wave function is given by $\psi(\mathbf{r},t) = \sqrt{\rho(r,t)}e^{il\phi}$.

Upon solving the resulting equations, we treat the condensate in the Thomas-Fermi approximation, both in the presence and in the absence of a vortex. Making this approximation is allowed, because the incoming atoms must always have an energy that is of the order of the chemical potential of the condensate. Therefore, effects of an exponentially decaying density profile at the boundary of the condensate, at an energy very close to the chemical potential, can safely be neglected. Thus, the density profile and the phase are given by

$$\rho(r) = \frac{\left[\mu - V_{\text{ext}}(r) - \frac{\hbar^2 l^2}{2mr^2}\right]}{T^{2\text{B}}}\Theta\left[\mu - V_{\text{ext}}(r) - \frac{\hbar^2 l^2}{2mr^2}\right] \tag{11a}$$

and

$$\chi(\phi) = \phi l, \tag{11b}$$

where $\Theta[x]$ denotes the Heaviside step-function. To lowest order, the change in the chemical potential due to the presence of a vortex can be neglected [15].

III. RESULTS

The method we use to calculate numerically the time evolution of an initial wave packet describing the incoming atom is based on Cayley's finite-difference representation of the Schrödinger equation [34],

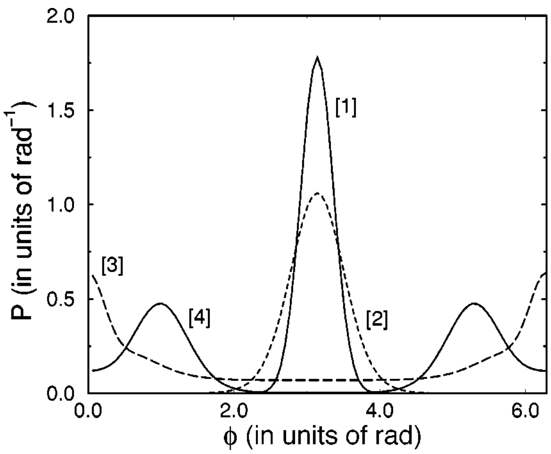

FIG. 1. The angular probability distribution $P(\phi,t)$ at four different times, in the absence of a vortex. At $t=0$, $P(\phi,t)$ is a Gaussian centered around $\phi=\pi$, and shown as line [1]. Line [4] is $P(\phi,t)$ after approximately half an oscillation, $t=3.5\omega_r^{-1}$. The short dashed line [2] and the long dashed line [3] are snapshots at intermediate times, $t=1.8\omega_r^{-1}$ and $t=2.0\omega_r^{-1}$, respectively. The calculation is performed for 2.6×10^6 ^{87}Rb atoms with a scattering length $a=5.77\times10^{-9}$ m in a trap with $\omega_r/2\pi=200$ Hz.

$$e^{iH\Delta t}\simeq\frac{1-iH\Delta t/2}{1+iH\Delta t/2}. \tag{12}$$

Here, Δt is the time step in the discretized Schrödinger equation. This algorithm is second-order accurate in time and unitary, i.e., the norm of the wave function is conserved up to computer accuracy.

The results of our numerical calculations are shown in Fig. 1 and Fig. 2, corresponding to a doubly spin-polarized ^{87}Rb condensate of 2.6×10^6 atoms without and with a vortex, respectively. In these calculations, the initial wave function of the incoming atom is given by the product of a Gaussian in both the r and the ϕ direction, centered around $r=r_0$ and $\phi=\pi$. In principle, we could have plotted the probability distribution $P(r,\phi,t)\equiv|f(r,\phi,t)|^2-|g(r,\phi,t)|^2$ as a function of r and ϕ. As time evolves, the hole part $g(r,\phi,t)$ of the atomic wave function becomes nonzero if the particle part $f(r,\phi,t)$ starts to overlap with the condensate. Also, the probability distribution $P(r,\phi,t)$ develops nodes in the radial direction. The wavelength of these nodes is a measure of the kinetic energy of the incoming particle. However, to present our data more comprehensively, we integrate with respect to r and denote the resulting angular probability distribution by $P(\phi,t)$. Note that $P(\phi,t)$ is normalized to one, and that this normalization is conserved numerically. The integrated distribution is shown at four different times, corresponding to $t=0$, approximately half a period of oscillation $t=3.5\omega_r^{-1}$, and two snapshots at $t=1.8\omega_r^{-1}$ and $t=2.0\omega_r^{-1}$. After half a period of oscillation, the incoming atoms have scattered off the condensate. The angle of the maximum scattering amplitude in Figs. 1 and 2 is roughly equal to what one would expect from considering the shadow cast by the condensate.

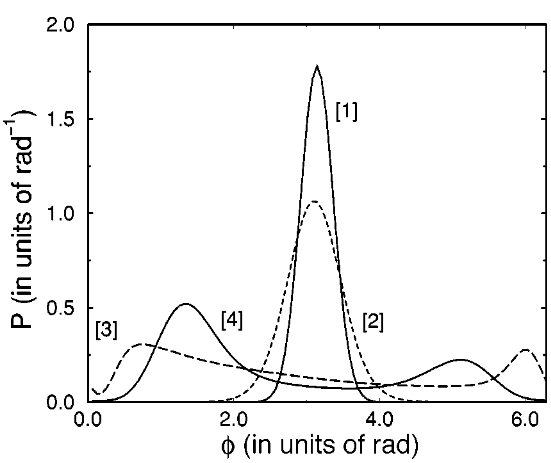

FIG. 2. The angular probability distribution $P(\phi,t)$ at four different times, in the presence of a vortex with $l=1$. The labeling of the curves, the times to which they correspond, and the parameters used are the same as in Fig. 1.

There are three main differences between Fig. 1 and Fig. 2. First, in Fig. 1, there is scattering in the forward direction, whereas in Fig. 2 there is almost no probability for scattering in this direction. Second, in Fig. 1, there is almost no scattering in the backward direction, whereas in Fig. 2 there is scattering in this direction. Third, and most important, in Fig. 1 the scattering is symmetric around $\phi=\pi$, whereas in Fig. 2 this symmetry is clearly broken. The first and second point are related to the presence of the vortex core, which causes the reflection in the forward direction to be decreased in favor of the backward direction. This has been checked numerically by inserting the core, but not the phase of the vortex, into the Bogoliubov–de Gennes equations and solving for the wave function of the scattering particle. The third point, however, cannot be related to the presence of the core, which is also rotationally symmetric. Instead, it is related to the phase of the condensate, which in the presence of a vortex indeed breaks rotational invariance. To make this point more clear, we present here a qualitative argument based on a semiclassical calculation of the Berry phase associated with an atom moving around the vortex. The calculation shows that there is an Aharonov-Bohm effect [35] in both the particle and the hole parts of the wave function, which causes the total atomic wave function to pick up a nontrivial phase factor.

The Berry phase θ associated with the spinlike nature of a two-component wave function is the phase picked up when the atom moves adiabatically along a classical trajectory C [36]. If the two-component wave function $|\psi(\mathbf{x})\rangle$ describes the ''spin'' degrees of freedom, it can be expressed as

$$\theta=i\int_0^t dt\langle\psi(\mathbf{x}(t))|\frac{\partial}{\partial t}|\psi(\mathbf{x}(t))\rangle$$
$$=i\int_C d\mathbf{x}\cdot\langle\psi(\mathbf{x})|\nabla|\psi(\mathbf{x})\rangle. \tag{13}$$

In the present case of the Bogoliubov–de Gennes equations, we thus get

$$\theta = i\int_C d\mathbf{x}\cdot[f^*(\mathbf{x})\nabla f(\mathbf{x}) - g^*(\mathbf{x})\nabla g(\mathbf{x})]$$

$$= -\frac{m}{\hbar}\int_C d\mathbf{x}\cdot \mathbf{v}_s(\mathbf{x})[|f(\mathbf{x})|^2 + |g(\mathbf{x})|^2]. \quad (14)$$

Here, the superfluid velocity $\mathbf{v}_s = \hbar\nabla\chi/m$, and the coherence factors $f(\mathbf{x})$ and $g(\mathbf{x})$ correspond to one of the local eigenfunctions of the Bogoliubov–de Gennes equations. In the presence of a vortex located at the origin with winding number l, and for a particle with an energy $E+\mu$ on a classically allowed circular trajectory at radius r, the Berry phase therefore becomes

$$\theta = -2\pi l\sqrt{1+\left[\frac{T^{2\mathrm{B}}\rho(r)}{E}\right]^2}. \quad (15)$$

This indeed shows that there is a nontrivial phase accumulated when the atom moves around the vortex, which causes the interference pattern to become asymmetric with respect to $\phi = \pi$. Note that it is essential that the particle and the hole components of the atomic wave function contribute with a different sign to the integral in Eq. (14). To understand the direction of the shift in the interference pattern, consider two interfering paths that enclose the vortex. Seen from $\phi = \pi$, one is passing on the left side, and the other is passing on the right side of the vortex. Because the accumulated phase for the contour $C = C_R - C_L$ is negative in the presence of a vortex with $l = 1$, we again get constructive interference if C_L becomes shorter and C_R longer. Therefore, the interference pattern for a particle coming from $\phi = \pi$ shifts towards $\phi = 0$, as seen numerically.

IV. CONCLUSIONS

In summary, we have presented a possible nondestructive method to detect the presence of a vortex. This is realized by colliding atoms with the Bose-condensed cloud that are identical to the atoms in the condensate. The most important signature of the vortex is an angular asymmetry in the scattered wave function, which we have related to the Aharonov-Bohm phases picked up by the particle and hole parts of the wave function. This is somewhat similar to a derivation of the Iordanskii force for phonons and rotons from an Aharonov-Bohm effect [37]. The angular asymmetry is also present in a weakly perturbed condensate, where it lifts the degeneracy of the collective modes with angular quantum numbers m and $-m$ [8,9,14,15,38]. We note that to use the detection mechanism in the way presented here, the trap should not rotate. Thus, after creating a vortex, the rotation of the trap must be stopped, in which case the vortex in principle becomes unstable. However, recent studies have shown that the time needed for the vortex to disappear from the condensate diverges with the number of particles in the condensate at least as $N_0^{2/5}$ [17,39]. As mentioned in the Introduction, the experimentally observed lifetimes are in the range from 400 to 1000 ms [19], which is typically much larger than ω_r^{-1} and thus, sufficient for our purposes. Hence, for large N_0 the instability of the vortex presents no difficulty for our detection scheme. Moreover, a detailed picture of the dynamics of the unstable vortex might be obtained by means of multiple collisions. In this paper, however, we considered only a time-independent vortex configuration, and the effect of vortex dynamics on the scattering wave function is a topic of future investigation.

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