# FREQUENCY INSTABILITY OF CRYOGENIC AND ROOM TEMPERATURE HYDROGEN MASERS.

J.M.V.A. Koelman, S.B. Crampton*, H.T.C. Stoof, O.J. Luiten**, and B.J. Verhaar

Department of Physics, Eindhoven University of Technology, Postbus 513 5600 MB Eindhoven, The Netherlands,
*Department of Physics and Astronomy, Williams College Williamstown, Massachusetts 01267,
**Natuurkundig laboratorium, University of Amsterdam 1018 XE Amsterdam.

ABSTRACT

Hyperfine interaction effects during spin-exchange collisions give rise to shifts of the frequency of oscillating hydrogen masers which not only depend on the atom density, but also on the hyperfine levels population distribution. At cryogenic temperatures the instabilities of these shifts are large compared to the potential thermal instabilities of liquid helium lined hydrogen masers.

## INTRODUCTION

Up to now, the hydrogen maser, with a state-of-the-art frequency instability below one part in $10^{15}$, is the most stable of all frequency standards. For various applications however, it is essential to have even better frequency standards. An illustrative example is the fact that available atomic clocks are not sufficiently stable to determine any irregularities in the period of the fastest millisecond pulsar dicovered [1].

Two years ago groups at MIT, UBC and Harvard reported [2] the first observations of maser oscillation at temperatures near 0.5 K using hydrogen masers with liquid helium coated walls. For this type of cryogenic hydrogen masers a frequency instability limit due to thermal fluctuations as low as two parts in $10^{18}$ was anticipated by Berlinsky and Hardy [3], leading to the exciting possibility for an improvement in the state-of-the-art of frequency stability with almost three orders of magnitude.

However, as we pointed out already briefly in a previous publication,[4] one may cast doubt on the realization of that large stability improvement because of frequency shifts associated with hydrogen atom spin-exchange collisions. In this contribution we investigate in detail the consequences of these frequency shifts on the stability of the cryogenic hydrogen maser.

## FREQUENCY SHIFTS IN OSCILLATING HYDROGEN MASERS

Spin-exchange collisions between the hydrogen atoms radiating on the low magnetic field $\Delta m_F=0$ hyperfine transition in a hydrogen maser frequency standard affect the maser frequency in two distinct ways. They directly shift the $\Delta m_F=0$ transition frequency $\omega$, and they broaden the atomic linewidth, which increases the frequency pulling due to cavity mistuning. In general, the direct shift is the sum of a shift $\Delta\omega_0$ due to wall collisions and other one-atom processes, and a shift $\Delta\omega_c$ due to interatomic collisions. The frequency shift due to cavity pulling is equal to the cavity mistuning parameter $\Lambda$ (approximately twice the ratio of cavity mistuning to cavity resonance width) times the total atomic linewidth $\Gamma$. In total we have for the maser frequency

$$\omega_m = \omega + \Delta\omega_0 + \Delta\omega_c + \Lambda\,\Gamma\,. \qquad (1)$$

The atomic linewidth can be written as the sum of a contribution $\Gamma_0$ not depending on the collision rate such as the line broadening due to the finite residence time of the atoms in the cavity, and a collision contribution $\Gamma_c$:

$$\Gamma = \Gamma_0 + \Gamma_c\,. \qquad (2)$$

Our quantum-mechanical calculation of the line shift due to spin-exchange collisions in an oscillating H-maser which takes into account the non-zero hyperfine energy levels splitting yields (for details of the derivation see Ref.4):

$$\Delta\omega_c = \Psi\,(1+\Lambda^2)\,\Gamma - \Omega\,\Gamma_c\,, \qquad (3)$$

with the frequency offset parameters $\Psi$ and $\Omega$ measuring the effect of spin-exchange collisions on the maser frequency. The parameter $\Psi$ is inversely proportional to the cavity quality factor and the cavity filling factor and also depends on temperature. The parameter $\Omega$ depends on temperature and, what is important, also on $\rho$: the fraction of the atoms in the two $m_F=0$ hyperfine states.

The usual theoretical treatment of hydrogen atom spin-exchange collisions [5], which treats the hyperfine energy levels during the collisions as degenerate, leads to analogous equations which however differ qualitatively from the above results as they lack the second term on the right hand side of Eq.3. Without the $\Omega$-term the maser frequency shift due to collisions is exactly proportional to the total linewidth as is the shift due to cavity pulling.[6] As in this case the linewidth $\Gamma$ is the only parameter which depends on the atom density $n_H$, tuning the cavity so that the oscillation frequency is the same at two different atom densities (spin-exchange tuning) is predicted to cancel the direct spin-exchange shift against the cavity mistuning shift and hence to leave the oscillation frequency independent of atom density: $\omega_m = \omega + \Delta\omega_0$.[7] As the atom density is

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The result of our more exact calculation of the direct shifts is that the simple proportionality of the spin-exchange shift to the atomic linewidth is broken. The hyperfine induced frequency shift parameter $\Omega$ depends on the level population sum $\rho$ which in general depends in a highly nonlinear way on the atom density. As a result, the standard spin-exchange tuning technique is sabotaged by hyperfine induced effects.

One might hope to be able to reduce the dependence of $\rho$ on $n_H$ by a proper setting of maser parameters which affect the hyperfine population dynamics, so that spin-exchange tuning again comes into play.[8] However, even if this is possible, hyperfine induced effects lead to important sources of frequency instabilities which strongly limit the stability to be reached with liquid helium lined H-masers. To show this we consider the dependence of the maser frequency on fluctuations of the atom density, the level population sum and the linewidth not due to collisions. Assuming $\rho$ to be decoupled from $n_H$, Eqs.(1-3) yield

$$\frac{\partial \omega_m}{\partial n_H} = [\Delta + \Psi(1+\Delta^2) - \Omega] \frac{\partial \Gamma_c}{\partial n_H} , \qquad (4)$$

$$\frac{\partial \omega_m}{\partial \rho} = [\Delta + \Psi(1+\Delta^2) - \Omega] \frac{\partial \Gamma_c}{\partial \rho} - \Gamma_c \frac{\partial \Omega}{\partial \rho} , \qquad (5)$$

$$\frac{\partial \omega_m}{\partial \Gamma_0} = [\Delta + \Psi(1+\Delta^2)] . \qquad (6)$$

The dependence on atom density can be removed by spin-exchange tuning the cavity, i.e. by setting $\Delta$ so as to have the same maser frequency at two different atom densities, in which case we are left with a maser frequency instability $\delta\omega_m$ due to fluctuations $\delta\rho$ in the level population sum and fluctuations $\delta\Gamma_0$ in the linewidth not due to collisions:

$$\delta\omega_m = \sqrt{\left[\frac{-\partial\Omega}{\partial\rho} \Gamma_c\right]^2 \delta\rho^2 + \Omega^2 \, \delta\Gamma_0^{\,2}} . \qquad (7)$$

At T=0.5K both $\Omega$ and $-\partial\Omega/\partial\rho$ reach their minimum values 0.056 and 0.022 respectively, for $\rho$=1. For $\Gamma_0 \simeq \Gamma_c \simeq 1s^{-1}$ these values imply a maximum allowed instability in the linewidth not due to collisions as low as $\delta\Gamma_0/\Gamma_0 = 3\times10^{-7}$ and a maximum allowed instability in the level population sum $\delta\rho/\rho = 6\times10^{-7}$ in order to achieve a frequency instability of 2 parts in $10^{18}$. It seems unlikely that all processes contributing to $\Gamma_0$ or $\rho$ can be kept stable within these limits.

At room temperature for $\rho=1$ we find $\Omega=2.7\times10^{-4}$ and $-\partial\Omega/\partial\rho=2.4\times10^{-4}$, both approximately two orders of magnitude smaller than at T=0.5K. A state-of-the-art frequency instability below 1 part in $10^{15}$ implies that for $\Gamma_0\simeq\Gamma_c\simeq 1s^{-1}$ the linewidth not due to collisions and the level population sum are both stable within two percent.

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