RADIATIVE CORRECTIONS TO VECTOR BOSON MASSES

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Weak and e.m. radiative corrections to vector boson masses are computed. Including corrections due to the presently known leptons and quarks, mass shifts of +3080 and +3310 MeV are obtained for the masses of the charged and neutral vector boson.

In the near future new machines may probably discover the vector bosons of weak interactions. Perhaps relatively accurate measurements of the masses may be made; if so, it may be possible, for the first time, to measure weak (and e.m.) radiative corrections, finite by virtue of the renormalizability [1] of the presently popular standard model [‡1]. Moreover, in an earlier paper [2], we have shown that radiative corrections to masses may constitute an important source of information concerning the mass spectrum above 100 GeV. It should be emphasized that this earlier calculation refers to masses seen as parameters in low-energy reactions; there is, in first instance, no direct relation to corrections to the actual vector boson masses, to be measured with future machines.

In another paper [3] the radiative corrections to μ-decay, the electric charge and the ratio of the $\bar{\nu}_\mu$e to the ν_μe scattering cross sections are reported. The available data on these processes may be used to fix the free parameters of the theory, including one-loop corrections, and after that further calculations result in finite, unambiguous answers. In particular one may compute the radiative corrections to the vector boson masses.

The result of such a calculation may conveniently be separated into two parts. First there is a contribution due to all kinds of effects on the above three processes. These include vertex contributions, bremsstrahlung etc., as reported in ref. [3]. Further there are contributions due to self-energy insertions in vector boson and photon propagators. In the first part we include self-energy diagrams involving vector bosons, photons and Higgs particles; self-energy diagrams involving fermions constitute the second part.

The total of corrections of the first part is gauge invariant by itself, quite small, but rather complicated. We therefore simply report the mass shift due to this part in the form of numbers: +120 and +140 MeV for M and M_0, respectively. As input we used the latest value for the weak mixing angle: $\sin^2\theta_\mathrm{w} = 0.238$ [4]. Furthermore we took arbitrarily $m_\mathrm{H} = 200$ GeV (m_H is the Higgs mass). For a Higgs mass of 3 GeV we find +20 and −30 MeV, and if the Higgs mass is 1000 GeV the values are +150 and +250 MeV. For large Higgs mass ($m_\mathrm{H} \gtrsim 200$ GeV) the mass shifts as a function of the Higgs mass are:

$$M: \quad +(g^2/384\pi^2)M\ln(m_\mathrm{H}^2); \qquad M_0: \quad +(g^2/384\pi^2)M_0(1+10s_\theta^2/c_\theta^2)\ln(m_\mathrm{H}^2),$$

where s_θ and c_θ denote the sine and cosine of the weak mixing angle. The coupling constant g is for the quoted value of s_θ^2 such that $g^2/\pi^2 = 0.039$. The above equations follow by noting that the Higgs mass dependence is only in S_+ and S_0 (see below), and by using the eqs. (11.5) of ref. [5]. Compared to the lowest-order vector boson masses of 76 500 and 87 640 MeV these corrections are of order 0.3%, which seems beyond practical interest. Actually, this weak dependence on the Higgs mass has been noted before, as an example of the screening theorem [2].

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[‡1] The "standard model" is defined to be the Weinberg model of leptons [7], enlarged to include quarks through the use of the GIM mechanism [8], and made renormalizable (i.e. also anomaly free) through the three-fold color degeneracy of the quarks [9].

Much more interesting from an experimental point of view are the contributions due to the self-energy insertions containing fermions. Let us call the ensemble of one lepton doublet and a three-fold color degenerate quark doublet a "flagpole" [2], and let us consider the contribution to the boson masses due to, say, the electron flagpole. We find:

$$\delta M^2 = \left[S_+(k^2) - \frac{c_\theta M^2 S_{\gamma 0}(k^2)}{s_\theta k^2} - \frac{M^2 S_\gamma(k^2)}{k^2} \right]_{k^2=0} - S_+(-M^2),$$

$$\delta M_0^2 = \frac{1}{c_\theta^2} \left[S_+(k^2) + \frac{s_\theta^2 - c_\theta^2}{c_\theta s_\theta} \frac{M^2 S_{\gamma 0}(k^2)}{k^2} - \frac{M^2 S_\gamma(k^2)}{k^2} \right]_{k^2=0} - S_0(-M_0^2),$$

and from this the mass shifts $0.5\,\delta M^2/M$ and $0.5\,\delta M_0^2/M_0$. In these expressions S_+, S_0, S_γ and $S_{\gamma 0}$ denote the contributions of the flagpole fermions to the W^+, W^0, γ and γ–W^0 self-energy parts as a function of the four-momentum k^2. The explicit formulae for these expressions are given in ref. [3]. Working out these equations we find the mass shifts as functions of the fermion and vector boson masses:

$$\delta M^2 = \frac{g^2}{16\pi^2} \left\{ \tfrac{4}{9} M^2 - \frac{1}{2M^2}(m_u^2 - m_d^2)^2 - \frac{1}{6M^2}(m_e^2 - m_\nu^2)^2 + \tfrac{3}{4}(m_u^2 - m_d^2) + \tfrac{1}{4}(m_\nu^2 - m_e^2) \right.$$

$$+ \left[\frac{m_u^2}{2M^2}(m_u^2 - m_d^2) - \tfrac{2}{3}M^2 - m_u^2 \right] \ln \frac{m_u^2}{m_d^2}$$

$$+ \left[\frac{m_\nu^2}{6M^2}(m_\nu^2 - m_e^2) - \tfrac{1}{3} m_\nu^2 \right] \ln \frac{m_\nu^2}{m_e^2} + \tfrac{3}{2} m_u^2 F_1(m_u, m_d) + \tfrac{1}{2} m_\nu^2 F_1(m_\nu, m_e)$$

$$+ \left[\frac{1}{2M^2}(m_u^2 - m_d^2)^2 - M^2 + \tfrac{1}{2}(m_u^2 + m_d^2) \right] B_{0\mathrm{r}}(-M^2, m_u, m_d)$$

$$\left. + \left[\frac{1}{6M^2}(m_\nu^2 - m_e^2)^2 - \tfrac{1}{3}M^2 + \tfrac{1}{6}(m_\nu^2 + m_e^2) \right] B_{0\mathrm{r}}(-M^2, m_\nu, m_e) \right\},$$

$$\delta M_0^2 = \frac{g^2}{\pi^2 c_\theta^2} \left\{ \frac{M^2}{108 c_\theta^2}(8c_\theta^4 - 10c_\theta^2 + 5) + \tfrac{1}{64}(3m_u^2 - 3m_d^2 + m_\nu^2 - m_e^2) - \frac{1}{32}\left(3m_u^2 + \frac{M^2}{3c_\theta^2}\right) \ln \frac{m_u^2}{m_d^2} \right.$$

$$- \frac{1}{32}\left(m_\nu^2 - \frac{M^2}{3c_\theta^2}\right) \ln \frac{m_\nu^2}{m_e^2} + \tfrac{1}{32}[3m_u^2 F_1(m_u, m_d) + m_\nu^2 F_1(m_\nu, m_e)]$$

$$+ \left[\frac{M^2}{288 c_\theta^2}(40c_\theta^2 - 32c_\theta^4 - 17) + \frac{m_u^2}{288}(80c_\theta^2 - 64c_\theta^4 - 7) \right] B_{0\mathrm{r}}(-M_0^2, m_u, m_u)$$

$$+ \left[\frac{M^2}{288 c_\theta^2}(4c_\theta^2 - 8c_\theta^4 - 5) + \frac{m_d^2}{288}(8c_\theta^2 - 16c_\theta^4 + 17) \right] B_{0\mathrm{r}}(-M_0^2, m_d, m_d)$$

$$\left. + \frac{1}{96}\left[\frac{M^2}{c_\theta^2}(12c_\theta^2 - 8c_\theta^4 - 5) + m_e^2(24c_\theta^2 - 16c_\theta^4 - 7) \right] B_{0\mathrm{r}}(-M_0^2, m_e, m_e) + \frac{1}{96}\left(m_\nu^2 - \frac{M^2}{c_\theta^2}\right) B_{0\mathrm{r}}(-M_0^2, m_\nu, m_\nu) \right\}.$$

The functions $F_1(m_1, m_2)$ and $B_{0r}(k^2, m_1, m_2)$ are given by:

$$F_1(m_1, m_2) = -y \ln [(y-1)/y] - 1, \quad F_2(m_1, m_2) = -y^2 \ln [(y-1)/y] - y - \tfrac{1}{2},$$

$$B_{0r}(k^2, m_1, m_2) = -\int_0^1 \mathrm{d}x \ln [(k^2/m_2^2)x(1-x) + (1 - m_1^2/m_2^2)x + m_1^2/m_2^2]$$

$$= -\ln(-k^2/m_2^2) + 2 - \sum_{i=1}^{2} [(1-x_i)\ln(1-x_i) + x_i \ln(-x_i)],$$

$$y = m_1^2/(m_1^2 - m_2^2), \quad x_{1,2} = \{k^2 + m_2^2 - m_1^2 \pm [(k^2 + m_1^2 + m_2^2)^2 - 4m_1^2 m_2^2]^{1/2}\}/2k^2 .$$

For future reference we included already the function F_2. The equations are in fact also correct including the imaginary part, relating to the decay width of the vector bosons. The above expressions are somewhat cumbersome, but simplify considerably in certain interesting cases.

First we note that the masses of the known fermions (including the as yet hypothesized top quark) are small with respect to the vector boson masses. In good approximation we may then take $B_{0r}(-M^2, m_1, m_2) = \ln(m_2^2/M^2) + 2 + \mathrm{i}\pi$. Neglecting all further terms that are small for small fermion mass we find ‡2 (ignoring now also the imaginary part):

$$\delta M^2 = -(g^2 M^2/\pi^2)\{\tfrac{5}{36} + \tfrac{1}{48}[2\ln(m_\mathrm{u}^2/M^2) + \ln(m_\mathrm{d}^2/M^2) + \ln(m_\mathrm{e}^2/M^2)]\},$$

$$\delta M_0^2 = -(g^2 M^2/\pi^2 c_\theta^2)[\tfrac{1}{108}(40c_\theta^4 - 50c_\theta^2 + 25) + \tfrac{1}{72}(8c_\theta^4 - 10c_\theta^2 + 5)\ln(m_\mathrm{u}^2/M_0^2)$$

$$+ \tfrac{1}{72}(2c_\theta^4 - c_\theta^2 + \tfrac{1}{2})\ln(m_\mathrm{d}^2/M_0^2) + \tfrac{1}{48}(4c_\theta^4 - 6c_\theta^2 + 3)\ln(m_\mathrm{e}^2/M_0^2)] .$$

For $c_\theta = 1$ these expressions are identical. The logarithms are rather large, and for instance, if we use the mass values 250 MeV for the quarks and 0.511 MeV for the electron then these equations lead to mass shifts of 1600 and 1700 MeV, respectively. Inclusion of the μ and τ flagpole and also the contribution of the first part with a Higgs mass of 200 GeV leads to mass shifts of 3080 and 3310 MeV for the charged and neutral vector boson, respectively. In this calculation we used the following mass values for the quarks: 250, 250, 1500, 300, 20000 and 5000 MeV for the up, down, charm, strange, top and bottom quark. To give an idea of the sensitivity to these numbers: if the up and down quark are taken to be 50 MeV then the mass shifts change by 300 and 290 MeV, respectively. Of course, using actual data on the hadron production by $\mathrm{e}^+\mathrm{e}^-$ one could pin down things much more precisely, but it is unlikely that this would change the quoted numbers by more than 300 MeV.

Another interesting case is when the fermion masses are larger than the W-masses. Then one may approximate:

$$B_{0r}(-M^2, m_1, m_2) = -F_1(m_1, m_2) + \frac{M^2}{m_1^2 - m_2^2}[F_1(m_1, m_2) - F_2(m_1, m_2)],$$

with F_1 and F_2 as defined before. Using this approximation one finds:

$$\delta M^2 = (g^2/24\pi^2)M^2, \quad \delta M_0^2 = (g^2/24\pi^2 c_\theta^2)M_0^2(1 - \tfrac{3}{2}\delta_\mathrm{q}^2/M_0^2 - \tfrac{1}{2}\delta_\ell^2/M_0^2),$$

‡2 Marciano [10] also quotes a formula for the charged W-mass shift containing logarithmic terms. His formula differs from ours due to a difference in the definition of s_θ. He defines c_θ as the ratio between the charged and neutral vector boson mass, while we define s_θ from the low-energy data, presumably close to the present experimental determination of this parameter. If this is taken into account then his results agree with ours. The numerical agreement (he also quotes 3 GeV) is somewhat accidental, because the difference between the log terms (about 50%) is almost compensated by the constant in our equation, and also because s_θ^2 happens to be about 0.25. We are indebted to Dr. Marciano for his comments on this point.

where δ_q and δ_ℓ are the mass differences between the quarks and between the leptons, respectively. The δ-dependent corrections to M_0^2 are just those found at low energies [2]. The fact that experimentally these low-energy corrections are small implies thus that the mass of the neutral W is not moved downwards because of heavy multiplets with mass differences. The δ-independent terms are not visible at low energies, and if there exist many heavy flagpole-type multiplets without mass differences then the W-masses could still shift upwards. This shift is, however, very small: each such flagpole contributes 60 MeV to M and 94 MeV to M_0. Such corrections would be hard to measure, and furthermore be difficult to distinguish from corrections due to heavy Higgs particles. In this context it may be mentioned that the weak and e.m. interactions would produce order α mass splittings between degenerate superheavy fermions that are not $SU_2 \times U_1$ singlets. Such mass splittings would be excluded for fermions over 20000 GeV by the low-energy data, but we must add immediately that heavy fermions are difficult objects to accommodate in the standard theory [6].

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References

[1] G. 't Hooft, Nucl. Phys. B35 (1971) 579.
[2] M. Veltman, Nucl. Phys. B123 (1977) 89;
M. Chanowitz, M. Furman and I. Hinchliffe, Phys. Lett. 78B (1978) 285;
M. Veltman, Talk at the Intern. Conf. on Photons and leptons (Fermilab, Aug. 23–29, 1979).
[3] M. Green and M. Veltman, Univ. of Utrecht preprint (Jan. 1980).
[4] I. Liede and M. Roos, Univ. of Helsinki preprint (Dec. 1979).
[5] M. Lemoine and M. Veltman, Nucl. Phys. (1980), to be published.
[6] H. Politzer and S. Wolfram, Phys. Lett. 82B (1979) 242; 83B (1979) 421, erratum;
P. Hung, Phys. Rev. Lett. 42 (1979) 873.
[7] S. Bludman, Nuovo Cimento 9 (1958) 433;
S. Glashow, Nucl. Phys. 22 (1961) 579;
S. Weinberg, discussion remark at the 1967 Solvay Conf. (Brussels), talk of H.P. Durr; Phys. Rev. Lett. 19 (1967) 1264.
[8] S. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.
[9] C. Bouchiat, J. Iliopoulos and Ph. Meyer, Phys. Lett. 42B (1972) 91.
[10] W. Marciano, Phys. Rev. D20 (1979) 274.