DOES QUANTUM CHROMODYNAMICS IMPLY CONFINEMENT?

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Abstract

The mechanism of permanent quark confinement in Quantum Chromodynamics is cast in a language that might enable one to perform more detailed dynamical calculations. The choice of gauge condition and the topological features added to the theory by fixing the gauge are crucial in this description. However, it also becomes clear that in spite of its popularity Quantum Chromodynamics has an "identity problem": the theory still has no sound mathematical basis. We speculate on remedies that one can envisage in the future.

1. INTRODUCTION

It is a long-standing problem how to devise a reliable method for computing physically observable quantities accurately in non-abelian gauge theories with strong interactions. One crucial step in solving this problem is to isolate the relevant dynamical variables at the critical distance scale (in quantum chromodynamics that is, of course, the hadronic mass scale). Approximation methods that are popular at present sometimes ignore some of these variables. In ordinary perturbation theory the compactness of the gauge group is not reflected, so that topologically non-trivial effects such as

instantons literally slip through the meshes. Theories based on instanton gases in their turn do not admit magnetic vortices, which we know to be crucial for understanding confinement. We do not as yet have a remedy for this situation. Nor do we have a clear insight on how seriously this drawback should be taken. Experiments suggest without much doubt that with "QCD" we are on the right track. But how well do we understand this theory?

In essence there are two problems. We list the two in the order that they are likely to be solved. The first is to obtain a good and reliable *qualitative* understanding. The observation that topological considerations are crucial here has been made a number of times [1,2]. In this paper we emphasize this and elucidate the present state of mind of the author (sects 2-5). Indeed these qualitative topological considerations yield a picture that could explain the experimental observations neatly. Very neatly, were it not that experimenters seem to be turning the clock backwards by observing some fractionally charged "quarks" [3]. To the best of my own understanding theory is unable to follow experiment here, in spite of some very ingenious attempts [4].

It is unlikely that the second problem on my list has anything to do with this fractional charge problem. It is the more subtle question of mathematical definition. Careful consideration of the perturbative theory, with its beautiful property of asymptotic freedom, strongly suggests that short-distance perturbation theory determines the dynamics completely, including the stronger interactions at long distances, because interaction at long distances could be viewed as an interplay of many weaker interactions at short distances. Renormalization group arguments suggest that one should choose as a bare coupling constant

$$g_B^2 \xrightarrow{M^2 \rightarrow \infty} \frac{16\pi^2}{\beta_0 \log(M^2/\Lambda^2) + (\beta_1/\beta_0) \log \log(M^2/\Lambda^2)} \quad (1.1)$$

where M is the cutoff with dimension of a mass and Λ is the QCD Λ parameter. β_0 and β_1 are the first two coefficients of the β -function, computed in diagrams with one and two loops, respectively. The other β -coefficients need not be known! They would only give corrections to g_B^2 that are so small that they can be absorbed in finite shifts of the Λ parameter. It would seem then that two-loop perturbation theory at small distances, together with the topologically defined instanton Θ parameter determine the dynamics of the theory. My problem is this: is that true? It is true up to any finite order in the perturbation expansion. But this expansion is known to diverge badly. This question has received relatively little attention in the literature. We will come back to it in section 6.

2. TOPOLOGY IN THE GAUGE CONDITION

The space of all vector potential configurations described by the set of functions $\{A_\mu^a(x)\}$ is essentially “flat” if all elements in this space are considered to be distinct and the usual topology for function spaces is introduced. A linear superposition of two non-vanishing vector potentials then gives a different vector potential.

However, if we introduce gauge invariance and all points on a gauge trajectory in our vector potential space are identified then we get an altogether different and highly non-trivial topology [5]. How is this reflected in the *physical* degrees of freedom? Can we localize topologically non-trivial features in space-time? This is the first problem of the two mentioned in the introduction.

We are interested in the question of what the *physical* degrees of freedom are. Therefore it is necessary to “fix the gauge”, that is, find a subset (section) of the space $\{A_\mu^a(x)\}$ that uniquely represents all gauge equivalence classes. Traditionally this is done by writing down a gauge condition such as

$$\partial_\mu A_\mu^a = 0 \quad (2.1)$$

(Lorentz gauge). Gauges such as these minimize the values of the vector potential fields (more precisely, the Lorentz gauge minimizes

$$W_1 = \int A_\mu^2 d^4x \quad (2.2)$$

under gauge rotations in Euclidean space). Therefore the Lorentz gauge is important to obtain ultra-violet convergent (“renormalizable”) perturbation expansions.

However, the trouble with the Lorentz gauge (2.1) is that at every point x the gauge function $\Omega(x)$ that transforms an arbitrary vector potential A_μ into one that satisfies (2.1) depends on field values at all other points. Thus spurious modes are created that propagate from one point to any other point, without having any direct physical interpretation. In perturbation theory these spurious modes (“ghosts”) can be dealt with completely [6]. Non-perturbatively, these modes obscure the topological structure of the theory. We will show this by choosing a “non-propagating” gauge; no ghost propagates, at the expense of a more violent ultraviolet behavior. Later one could choose to tame the ultraviolet divergences by allowing some very heavy ghost to occur (thereby introducing only some very short range propagation of non-physical gauge effects).

Since it is of foremost importance to understand pure quantum chromodynamics (i.e. without additional scalar fields), the gauge must be fixed by the vector field itself.

And, since we wish to have the relevant gauge transformations $\Omega(x)$ determined by fields at the same point $A_\mu(x)$ only, we should use a field or field combination that transforms under Ω non-trivially, but without the derivative term (the term containing $\partial_\mu \Omega$ or $\partial_\mu \Omega^{-1}$). Such a field is the covariant curvature $G_{\mu\nu}^a$ or $(G_{\mu\nu}^{ij} \equiv \lambda_{ij}^a G_{\mu\nu}^a)$. So a good candidate for a gauge condition is the gauge that brings one of the components, say G_{12}^{ij} , into diagonal form:

$$X^{ij} = \lambda^{(i)} \delta_{ij}, \quad (2.3)$$

where

$$X^{ij} = G_{12}^{ij}. \quad (2.4)$$

We will choose the λ^i to be ordered:

$$\lambda^{(1)} > \lambda^{(2)} > \dots > \lambda^{(N)}. \quad (2.5)$$

But because this X is not Lorentz-invariant, one breaks Lorentz-invariance in the same process, which is usually not very desirable.

So instead of (2.4) we could substitute for X in (2.3) some other field combination, such as

$$X^{ij} = (G_{\mu\nu} G_{\mu\nu})^{ij} \quad (2.6)$$

or

$$X^{ij} = (G_{\mu\nu} D_\alpha^2 G_{\mu\nu})^{ij}. \quad (2.7)$$

The problem with Eq. (2.6) is that it does not fix the gauge if the gauge group is $SU(2)$, because then Eq. (2.3) is trivially satisfied, and Eq. (2.7), though Lorentz-invariant, may cause problems because of its complicated form (D_α is the covariant derivative).

An intermediate gauge choice could be

$$\alpha D_\mu^o A_\mu^{ch} + i\beta [X^{ch}, X^o] = 0, \quad (2.8)$$

where the superscript "o" stands for the diagonal part of X only, and the superscript "ch" for the off diagonal part of X only. D_μ^o is the covariant derivative with respect to

the diagonal part of A_μ only. α and β are (positive) gauge parameters. Eq. (2.8) corresponds approximately to extremizing

$$W_2 = \int d^2x \text{Tr} \{ g\alpha (A_\mu^{ch})^2 + \beta (X^{ch})^2 \} \quad (2.9)$$

under gauge rotations in Euclidean space.

The gauges Eq. (2.3) and Eq. (2.8) have in common that they leave the largest abelian subgroup unbroken. This is the group of $\Omega(x)$ that commute with X and therefore in general with itself (if the λ do not coincide). Indeed, in any generalization of the above procedure (for other gauge groups, say) one should choose the gauge condition such that one of the largest abelian subgroups remain unbroken.

3. TOPOLOGICAL GAUGE ARTIFACTS

In the gauge Eq. (2.3) but also in the hybrid gauge Eq. (2.8) one finds that the Euclidean functional integral gets contributions from topologically stable singular field configurations. These singularities occur where and when two eigenvalues of λ^i of X coincide. At such points the gauge is not properly fixed. Let us demonstrate this explicitly of the gauge Eq. (2.3) when the gauge group is $SU(2)$.

The gauge quantum numbers of X^{ij} are those of an isovector (X^1, X^2, X^3): $X^{ij} \equiv X^a \tau_{ij}^a$, and the gauge Eq. (2.3), together with Eq. (2.5) corresponds to

$$X^a = (0, 0, X) ; X > 0, \quad (3.1)$$

except when $\lambda^{(1)} \rightarrow \lambda^{(2)}$. Then X^{ij} is an isoscalar and in Eq. (3.1) $X^a \rightarrow 0$; $X \rightarrow 0$.

The unbroken subgroup mentioned in the end of the previous section is $U(1)$, the set of rotations about the X -axis. However at those points in space-time where the two eigenvalues coincide we have $X = 0$, and there our gauge condition leaves an invariance group $SU(2)$. Now prior to fixing the gauge, our X -vector was just an arbitrarily chosen field, whose zeros had no special significance. The location of those zeros is determined by the three equations $X^a = 0$; therefore they are in general one dimensional subspaces of $\mathcal{R}^4$, or pointlike in $\mathcal{R}^3$. After fixing the gauge, these point-particle-like structures correspond to singularities in our fields. Singularities, because the gauge rotation $\Omega(x)$ that restores Eq. (3.1) in their vicinity gets an infinite gradient at those points. So, the vector fields $A_\mu(x)$ that transform with $\partial_\mu \Omega$ become singular.

The significance of these special points with respect to the $U(1)$ ("electromagnetic")

subgroup of the gauge group is easy to deduce from our study of their cousins in the 1972 Georgi-Glashow model [7]. That is a model in which the vacuum expectation value $\langle X^a \rangle = F\{0, 0, 1\} \neq 0$, which implies that the $U(1)$ invariance of the X -gauge is also an invariance of the vacuum, so that the $U(1)$ photons are physical. It has been established [8], and we do not repeat those arguments here, that the zeros of X in that model carry two Dirac units of pure magnetic monopole charge. In our system also, the zeros of X are to be identified with magnetic monopoles. However they might undergo Bose-condensation which gives the system a long distance structure entirely different from that of the 1972 Georgi-Glashow model.

We can readily generalize our arguments for the case that the original gauge group was $SU(3)$. Our X field is an $SU(3)$ octet, and its diagonalization in the generic case leaves a subgroup $U(1) \times U(1)$ invariant. Indeed this is again the largest abelian subgroup. It will be easier to represent this as

$$U(1)^3/U(1). \quad (3.2)$$

Each gluon carries one positive $U(1)$ charge and one negative $U(1)$ charge:

$$Q^{(i)} = \pm (g, -g, 0), \quad (3.3.a)$$

$$\text{or } \pm (g, 0, -g), \quad (3.3.b)$$

$$\text{or } \pm (0, g, -g). \quad (3.3.c)$$

The two neutral gluons correspond to $U(1)$ photons. Notice that $\sum_i Q^{(i)} = 0$, so that one $U(1)$ drops out, as indicated in Eq. (3.2).

Again, we have singularities if two eigenvalues of X coincide. That could either be the first two or the last two but not the first and last because the λ 's were ordered. These singularities again turn out to be particle-like and carry the following magnetic charges.

$$M^{(i)} = \pm \left(\frac{2\pi}{g}, -\frac{2\pi}{g}, 0 \right), \quad (3.4.a)$$

$$\text{or } \pm \left(0, \frac{2\pi}{g}, -\frac{2\pi}{g} \right). \quad (3.4.b)$$

The combination analogous to Eq. (3.3.b) does not occur. Again, $\sum_i M^{(i)} = 0$.

Our claim is that magnetic monopole singularities will always arise whenever one attempts to fix the gauge locally, using only the fields occurring in the pure gauge theories. They even persist in smoother gauges such as Eq. (2.8). See [9] where also the effects of instantons are taken into account.

4. PHANTOM SHEETS

Let us once again consider the intermediate gauge Eq. (2.8). What kind of ghost should one expect? If β is large enough then according to Eq. (2.9), X^{ch} tends to become small. After an infinitesimal gauge transformation we have approximately:

$$X^{ch} \rightarrow i [\Lambda^{ch}, X^o], \quad (4.1)$$

and

$$A_\mu^{ch} \rightarrow -D_\mu \Lambda^{ch}. \quad (4.2)$$

Substituting this in Eq. (2.9) we get a Lagrangian for the ghosts:

$$\mathcal{L} = g\alpha (D_\mu \Phi^{ch})^2 + \beta |[\Phi^{ch}, X^o]|^2 + \text{interaction terms}. \quad (4.3)$$

From this we read off that the mass of the ghost will be

$$m_{gh} = \mathcal{O}[\sqrt{g\beta/\alpha} |\lambda^{(i)} - \lambda^{(j)}|], \quad (4.4)$$

where the $\lambda^{(i)}$ are the eigenvalues of X . As long as the coupling constant $g \neq 0$, we expect that this mass will tend to infinity if β tends to infinity. In the limit $\alpha \rightarrow 0$ the non-propagating gauge should be reached.

This gauge is unstable against formation of monopole singularities because at such singularities W_2 , Eq. (2.9) still converges. However, a new topological structure can now appear: "phantom sheets". This is because the system will not always keep the eigenvalues $\lambda^{(i)}$ of X^o ordered. We get various regions of space-time where their order will be different. These regions will be separated by domain walls much like Bloch walls. Let us take a closer look at these walls.

Consider for simplicity again the case $SU(2)$. We write

$$X = \lambda + a_k \sigma_k. \quad (4.5)$$

Here σ_k are the Pauli matrices. We are far away from any singular points, so

$$|\vec{a}| = a \neq 0. \quad (4.6)$$

We take m_{gh} to be large with respect to the scale in which the physical fields vary. Then we are close to a pure gauge transformation of the approximately constant field configuration,

$$X' = \lambda + a'_3 \sigma_3 ; A'_\mu = 0, \quad (4.7)$$

where a'_3 and A'_μ are space time independent. This configuration is transformed by a gauge transformation $\Omega(x)$ such that at $x_3 \gg 0$ we have

$$X \equiv \Omega X' \Omega^{-1} = \lambda + a'_3 \sigma_3 ; \quad A_\mu \equiv ig^{-1} \Omega \partial_\mu \Omega^{-1} = 0 \quad (4.8)$$

and at $x_3 \ll 0$ we have

$$X = \lambda - a'_3 \sigma_3 ; \quad A_\mu = 0. \quad (4.9)$$

Ω must be of the form

$$\Omega = \exp(i\omega \sigma_1), \quad (4.10)$$

where ω depends on x_3 , and

$$\begin{aligned} W_2 &= \int d^2x \operatorname{Tr} [-g^{-1} \alpha \{ (\Omega \partial_\mu \Omega^{-1})^{ch} \}^2 + \beta \{ (\Omega X' \Omega^{-1})^{ch} \}^2] \\ &= 2 \int d^2x \{ \beta a^2 \sin^2 2\omega + g^{-1} \alpha (\partial_\mu \omega)^2 \}. \end{aligned} \quad (4.11)$$

This is the Lagrangian for a sine-Gordon equation, and the boundary conditions Eq. (4.8) and Eq. (4.9) are those of the sine-Gordon soliton. The contribution of the sheet to W_2 is

$$W_2 = 4a \sqrt{\alpha/\beta} g^{-3/2} \int d^2x, \quad (4.12)$$

where the integral is over the sheet. The thickness of the sheet is of order

$$a^{-1} \sqrt{\alpha/\beta} g^{-1/2} = \mathcal{O}(m_{gh}^{-1}). \quad (4.13)$$

We may safely assume that the phantom sheets contribute to W_2 by an amount proportional to their area. Since they have no natural boundary they only come in the form of bubbles, and minimizing W_2 will correspond to minimizing the area of these bubbles, which will therefore never grow to substantial sizes. So, in the bulk of space-time, with only small exceptional regions, we may assume that the Λ 's are ordered in the same way as in the ghost-free gauge Eq. (2.3), see Eq. (2.5). Monopole singularities will occur in this gauge as much as in Eq. (2.3). Because of the above, in the case of larger gauge groups, the magnetic charges will be consecutive (in the sense of Eq. (3.4.a)) in the "regular" regions of space time, but non-consecutive

charges might show up occasionally, inside a bubble. We see that although gauges of the type Eq. (2.8) might be easier to handle in perturbation theory, they give rise to a more complicated topology.

We conclude that perhaps the most crucial physical dynamical variables that govern the strong-interaction region of a non-Abelian theory may be obtained by first fixing the non-Abelian part of the gauge symmetry. We get an Abelian theory with magnetic monopoles. The short range non-electromagnetic interactions between the electrically charged particles and the monopoles may perhaps be computable by "ordinary" perturbative techniques.

5. CONFINEMENT

Let us now concentrate on the quantized theory of electromagnetic fields, electrically charged particles and magnetically charged particles. In the simplest case Dirac's condition [10] for the charge quanta is:

$$QM = 2\pi n, \quad (5.1)$$

with n an integer. More precisely, and in the case that there are more than one kind of electromagnetisms (in the case that the original gauge group was $SU(3)$ or larger),

$$\sum_i (Q_i^{(1)} M_i^{(2)} - Q_i^{(2)} M_i^{(1)}) = 2\pi n^{(1,2)} \quad (5.2)$$

for any pair of particles (1) and (2). So either the Q 's, or the M 's, or both, must be large. This makes quantization extremely difficult.

We now notice that the *classical* theory has an invariance under rotations of the type

$$\begin{aligned} Q_i &\rightarrow Q_i \cos\Theta_i + M_i \sin\Theta_i, \\ Q_i &\rightarrow M_i \cos\Theta_i - Q_i \sin\Theta_i \end{aligned} \quad (5.3)$$

as far as the electromagnetic interactions are concerned. But we also know that the quantized interactions of the electromagnetic fields are completely determined by the classical theory. So the invariance Eq. (5.3) is an exact invariance of the quantized Maxwell system as well. Notice, indeed, that Eq. (5.2) is invariant.

Now Eq. (5.2) is not exactly the same as Eq. (5.1) if there is only one kind of charge. Eq. (5.2) allows monopoles themselves to carry fractional electric charge. Our monopoles will carry fractional electric charge if the so-called instanton angle differs from zero. Let us only concentrate on the case that this angle equals zero. Then we will choose a transformation Eq. (5.3) with $\Theta_i = 90^\circ$. After that we will do ordi-

nary field theory. This implies that, at least to some extent, the original electric charges, now magnetic charges, are ignored. The fundamental assumption in quantum-chromodynamics is that that is reasonable.

Another assumption goes even farther than that. We assume that the vacuum becomes a superconductor with respect to the new "electric" charges. That is not so unreasonable. Perturbation theory tells us that the Q_i become large at the hadronic distance scale. According to Eq. (5.2) then, the magnetic charges become weak. One can imagine very well that this will bring their chemical potential to the level of Bose condensation. The mechanism for Bose condensation is in no way different from the one in ordinary superconductors, or, in the language of particle physicists, the Higgs model. However, whether or not Bose condensation does take place in our case, depends on the dynamics. We still have great difficulties in doing reliable computations here: at small distance scales the Q_i are small, so that perturbation expansions with respect to the Q_i must be performed, and at large distance scales the M_i are small. It is in the transition region where both are fairly large that we do not know how to extract reliable results.

There is no doubt that *if* the original magnetic charges become superconducting, quarks, or any other objects with color, will be permanently confined. The argument is simple. After our transformation Eq. (5.3), with $\Theta = 90^\circ$, these objects become magnetically charged. But their magnetic flux is expelled from the superconductor by the Meissner effect: The flux lines are squeezed into narrow tubes (the Abrikosov fluxes in a superconductor). These tubes are the strings that tie quarks together permanently. We conclude that we are perfectly able to understand permanent quark confinement qualitatively. It is "reasonable" for the system to condense into a "quark confining mode". What is still lacking is a convincing proof that the dynamics is indeed such that this condensation takes place. Model dependent calculations, such as the Monte Carlo computations on lattice gauge theories give strong evidence in favor of the above picture. However the next section should be a warning against too rapid conclusions from those results.

6. IS QUANTUM CHROMODYNAMICS A THEORY?

All we really understand of quantum chromodynamics is its perturbation expansion. For calculations at small distance scales the expansion parameter is small, so the small distance structure of the theory seems to be well understood. Since we expect that the large distance interactions can be seen as an interplay of small distance interactions, one might conclude that such an asymptotically free theory has a sound and unique

mathematical basis. This however is not true, as yet. Sound "constructive field theories" have been discovered in two and three space-time dimensions, but not in four. One fundamental obstacle seems to be that in four space-time dimensions asymptotic freedom is only logarithmic, which is too slow. Another is a divergence in the Borel resummation procedure due to instantons. There are three different attitudes a physicist can have regarding these problems, besides ignoring them. One, obviously, is assuming that although our present day mathematics is not yet capable of producing accurate numbers in QCD, improvement will be possible, so that, eventually, one will be able to prove that all or some asymptotically free field theories in four dimensions are well defined and unique. Numbers such as the ratio between the proton mass and the string constant can perhaps not in practice but in principle be computed up to any desired accuracy.

What if this is not so? One then must realize that since the floating coupling constant depends only logarithmically on the distance scale, one might sooner or later have to deal with quantum gravity effects. That is possibility number two, and if this is inevitable then there seems to be little chance that we will be able to handle QCD within our lifetime.

A third possible attitude is that including quantum gravity can be avoided, but quantum chromodynamics all by itself is not good enough. We must replace it by a theory with perhaps more additional fields, in such a way that the diseases can be cured. Indeed we have in mind that such a procedure might exist. In the following section we sketch the argument. The assumption that quantum chromodynamics all by itself is not good enough implies for instance that the presently popular lattice approximations will eventually run into trouble. Physical quantities computed this way will not become independent of the lattice artifacts of the models, even in the limit (lattice parameter) $\rightarrow 0$. Probably computations at present are not sensitive enough to reveal this. Replacement of QCD by the model(s) in the next section might be comparable to the replacement of non-renormalizable theories by renormalizable ones: the latter are more valuable because computations can be done more accurately, even if the former are not evidently wrong. The price is often the introduction of new degrees of freedom.

7. ASYMPTOTICALLY FREE $1/N$ EXPANSIONS

Let us try to change the theory such that the coupling constant goes to zero *faster* than logarithmically. Thus we imagine that the β -function increases with energy. The first coefficient is the only one we have to look at. It is proportional to N , for $SU(N)$

theories. So it is natural to suggest that N increases indefinitely with energy. Does this indeed improve the theory? This is not known, but there is a fair chance that such models will be much better from a mathematical point of view than ordinary QCD with $N=3$.

What we can say with some confidence is that the $N \rightarrow \infty$ limit of $SU(N)$ gauge theories is unique. As is well known [11], this limit is governed by planar Feynman diagrams. These can still become infinitely complicated, and no analytic solution of this limit theory is known. But the diseases of the finite N theory do not arise here. In the limit, g^2N is kept fixed whereas instantons carry an action proportional to $1/g^2$, without any further N -dependence. Consequently, bothersome instanton singularities will disappear from the Borel sum. In terms of g^2N , the theory is still only logarithmically asymptotically free, but on the other hand the expansion in terms of g^2N is much better behaved. The total number of different planar diagrams with L loops and E external lines is of order C^L , where C depends on details of the theory [12]. At finite N this number goes as $C^L L!$, a fundamentally faster divergence. What we claim is now that, in the presence of a sufficiently effective infrared cut-off, the Dyson-Schwinger equations determine all Green functions of the planar theory uniquely. A proof, or at least arguments that favor this claim strongly, will be published elsewhere.

The infrared cut-off can be realized by enclosing the system in a sufficiently small box. The complete theory is then obtained by gluing boxes together. Planarity seems to be crucial for the argument. An important lemma that we can prove states that the subset of all ultraviolet *convergent* planar diagrams in Euclidean space converges with a finite radius of convergence when summed. That lemma even holds without infrared cut-off.

Given that the $N \rightarrow \infty$ theory is fine, what remains to be done is to invent a "tumbling mechanism" enabling us to go gradually from $N = 3$ to $N \rightarrow \infty$. What first springs to mind is that the $SU(N)$ symmetry breaks down spontaneously, step by step into $SU(3)$. Of course the symmetry breaking mechanism could be dynamical in nature, but a model can be designed where ordinary scalar Higgs fields are responsible. This is a super-symmetric model with an $SU(N)$ gauge super-multiplet and roughly $2N$ super-fields in the elementary representation of $SU(N)$. Due to super-symmetry it is still asymptotically free. Then mass terms are added that ignore super-symmetry entirely. They vary wildly in order of magnitude. One obtains an infinite series of thresholds $M_3, M_4, \dots, M_N, \dots$ where the apparent symmetry is raised from $SU(3)$ to $SU(4)$ to $\dots$ to $SU(N)$, etc.

Past experience gives us reason to hope that the spontaneous symmetry breaking will not upset the superior convergence properties of this theory, but a lot of work still must be done to sort this out.

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