ONE-LOOP CORRECTIONS TO $e^+e^- \rightarrow e^+e^-$ IN THE WEINBERG MODEL*

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Radiative corrections to Bhabha scattering are calculated in the simplest example of non-Abelian gauge theories. A detailed analysis of the higher-order effects is presented and the total differential cross section including weak corrections is evaluated at different angles in an energy range up to 70 GeV per beam. The effects of weak interactions appear to be negligible up to the production of the neutral vector boson.

1. Introduction

Since the observation of neutral currents in neutrino experiments and the apparent verification of the charm hypothesis, a unified gauge theory of weak and electromagnetic interactions is the most serious candidate to represent our present understanding of these phenomena.

On the other hand at the presently available energies there is evidence neither for the existence of the intermediate vector bosons nor of the other particles predicted by the various models. We can only say that somewhere at high energies the conventional four-fermion theory of weak interactions must suffer large corrections coming from its non-renormalizable character as well as from the predicted violation of the unitarity bound. In this context high-energy e^+e^- annihilation can provide a test for the properties of weak interactions since these interactions are expected to be dominant in some energy range whatever the mass of the vector bosons.

Up to now experimental data [1] on the reactions

$$e^+e^- \rightarrow \mu^+\mu^-, \quad (1)$$

$$e^+e^- \rightarrow e^+e^-, \quad (2)$$

have been in perfect agreement with the predictions of pure QED [2–4] including

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radiative corrections, and weak interaction effects were negligible in the energy range explored. In the future, very high energy electron-positron machines [5] may allow detailed study of these processes and weak effects, in particular weak radiative corrections, may become important. For this reason we have undertaken an investigation of reaction (2).

Before describing our results some remarks are required. The problem of calculating radiative corrections due to weak interactions was solved in principle since the proof [6] of the renormalizability of spontaneously broken gauge theories was given, but the complicated structure of interactions arising from the Lagrangian in the renormalizable gauge (even in the simplest case of the Weinberg model [7]) is certainly quite different from the simplicity of pure QED and in addition there are serious problems connected with the evaluation of the various Feynman integrals. A suitable calculation scheme has been proposed recently [8] in order to handle the one-loop integrals in general cases. The full derivation of the expressions in terms of Spence functions has been carried out and the complications due to the large spectrum of energies and masses one has to explore have been treated with the help of a high-precision program for the numerical evaluation of the analytical expressions. The results of ref. [8] furnish the mathematical background on which this work is founded. Another difficulty is that realistic calculations cannot be performed without introducing some *ad hoc* choices of the various physical quantities of the Weinberg model. We have taken for the weak mixing angle the value [9]

$$\sin^2 \theta_w \sim 0.22, \quad (1.1)$$

from which the value of the $SU(2)_L$ coupling constant has been derived according to the model Lagrangian relation

$$\frac{g^2}{4\pi} = \frac{\alpha}{\sin^2 \theta_w}, \quad (\alpha \text{ being the fine structure constant}). \quad (1.2)$$

The mass of the charged vector boson has been deduced from the phenomenological relation of μ -decay

$$\frac{g^2}{8M^2} \sim \frac{G_F}{\sqrt{2}}, \quad (G_F \text{ being the Fermi coupling constant}), \quad (1.3)$$

while the mass of the Higgs particle has been chosen quite large (the results are not sensitive to this mass).

The outline of this paper is as follows. In sect. 2 the lowest-order cross section for Bhabha scattering in the Weinberg model is given; in sects. 3–9 the higher-order corrections are discussed, while in sect. 10 we present our results. In appendix A the relevant Feynman rules are listed. In appendix B we give a short review of the methods of ref. [8]. In appendices C and D we have concentrated some technical details connected with the various self-energy parts and the bremsstrahlung.

The following conventions and notations are used: metric, γ -matrices and spinors as in ref. [10]; m = electron mass, m_μ = muon mass, M = charged vector boson mass, M_0 = neutral vector boson mass, M_z = Higgs particle mass, $s_w = \sin \theta_w$, $c_w = \cos \theta_w$, E = beam energy, θ = scattering angle. A list of the symbols used in the various diagrams is given in appendix A.

The momenta in all the diagrams are taken inwards, moreover K_1 = incoming electron four-momentum, K_2 = incoming positron, Q_1 = outgoing electron, Q_2 = outgoing positron, with $K_1 + K_2 + Q_1 + Q_2 = 0$. The relations for the invariants in the c.m. ($S = -(K_1 + K_2)^2 = 4E^2$, $T = -(K_1 + Q_1)^2 = 2(E^2 - m^2)(\cos \theta - 1)$, $U = -S - T + 4m^2$) in the limit of vanishing electron mass are

$$S = 4E^2, \quad T = 2E^2(\cos \theta - 1), \quad U + S + T = 0.$$

We will often use the notation “S- or T-channel” in order to define a diagram (or set of diagrams) having a spinorial path connecting respectively electron-positron or electron-electron external lines.

The masses of the leptons are neglected everywhere in the numerators of the various amplitudes (except in the self-energy of the electron where also terms proportional to $1/m^2$ appear). For this reason also their couplings to the scalar fields (proportional to the ratio m/M) have not been introduced.

2. Lowest-order cross section

Four diagrams contribute to Bhabha scattering at the tree level, they are shown in fig. 1. The corresponding expression for the amplitude is

$$\begin{aligned} M^0 = (2\pi)^4 ig^2 \{ & \bar{v}(K_2) \gamma_\mu u(K_1) \bar{u}(-Q_1) \gamma_\mu v(-Q_2) s_w^2 D(-S) \\ & + \bar{v}(K_2) \gamma_\mu (a + b\gamma_5) u(K_1) \bar{u}(-Q_1) \gamma_\mu (a + b\gamma_5) v(-Q_2) \Delta(-S) \\ & - \bar{u}(-Q_1) \gamma_\mu u(K_1) \bar{v}(K_2) \gamma_\mu v(-Q_2) s_w^2 D(-T) \\ & - \bar{u}(-Q_1) \gamma_\mu (a + b\gamma_5) u(K_1) \bar{v}(K_2) \gamma_\mu (a + b\gamma_5) v(-Q_2) \Delta(-T) \}, \end{aligned} \quad (2.1)$$

where

$$D(x) = \frac{1}{x}, \quad \Delta(x) = \frac{1}{x + M_0^2}, \quad (2.2)$$

Fig. 1. Bhabha scattering in the tree approximation.

and

$$a = \frac{4s_w^2 - 1}{4c_w}, \quad b = \frac{-1}{4c_w}. \quad (2.3)$$

It is useful to introduce the chiral coupling constants

$$g_L = a + b, \quad g_R = a - b, \quad (2.4)$$

the following combinations of propagators

$$\begin{aligned} \Delta_{LL}(x) &= s_w^2 D(x) + g_L^2 \Delta(x), \\ \Delta_{LR}(x) &= \Delta_{RL}(x) = s_w^2 D(x) + g_R g_L \Delta(x), \\ \Delta_{RR}(x) &= s_w^2 D(x) + g_R^2 \Delta(x), \end{aligned} \quad (2.5)$$

and the left- and right-handed currents

$$\begin{aligned} J_{1\mu}^{L,R}(-S) &= \bar{v}(K_2) \gamma_\mu (1 \pm \gamma_5) u(K_1), \\ J_{2\mu}^{L,R}(-S) &= \bar{u}(-Q_1) \gamma_\mu (1 \pm \gamma_5) v(-Q_2), \\ J_{1\mu}^{L,R}(-T) &= \bar{u}(-Q_1) \gamma_\mu (1 \pm \gamma_5) u(K_1), \\ J_{2\mu}^{L,R}(-T) &= \bar{v}(K_2) \gamma_\mu (1 \pm \gamma_5) v(-Q_2); \end{aligned} \quad (2.6)$$

moreover,

$$\begin{aligned} J_{LL}(x) &\equiv J_{1\mu}^L(x) J_{2\mu}^L(x), \\ J_{LR}(x) &\equiv J_{1\mu}^L(x) J_{2\mu}^R(x), \\ J_{RL}(x) &\equiv J_{1\mu}^R(x) J_{2\mu}^L(x), \\ J_{RR}(x) &\equiv J_{1\mu}^R(x) J_{2\mu}^R(x). \end{aligned} \quad (2.7)$$

In terms of the new variables the lowest-order amplitude has the following form:

$$\begin{aligned} M^0 &= (2\pi)^4 i_4^{\frac{1}{2}} g^2 \{ J_{LL}(-S) \Delta_{LL}(-S) + J_{RR}(-S) \Delta_{RR}(-S) \\ &\quad + (J_{LR}(-S) + J_{RL}(-S)) \Delta_{RL}(-S) - (S \leftrightarrow T) \}. \end{aligned} \quad (2.8)$$

From (2.8) one can then easily derive the differential cross section

$$\begin{aligned} \frac{d\sigma^0}{d\Omega} &= \left[\frac{g^2}{4\pi} \right]^2 \frac{1}{4S} \{ U^2 [\Delta_{LL}(-S) + \Delta_{LL}(-T)]^2 + U^2 [\Delta_{RR}(-S) + \Delta_{RR}(-T)]^2 \\ &\quad + 2T^2 \Delta_{RL}^2(-S) + 2S^2 \Delta_{RL}^2(-T) \}. \end{aligned} \quad (2.9)$$

By putting $g_L = g_R = 0$ and $g^4 s_w^4 / (4\pi)^2 = \alpha^2$, expressing S , T , U in terms of the beam energy and of the scattering angle one gets the usual high-energy limit of Bhabha scattering in QED [4]:

$$\frac{d\sigma^0}{d\Omega} = \frac{\alpha^2}{16E^2} \frac{[9 + 6 \cos^2 \theta + \cos^4 \theta]}{(1 - \cos \theta)^2}. \quad (2.10)$$

The decomposition of the various amplitudes in left- and right-handed parts is worthwhile in order to handle the many terms coming from all one-loop diagrams; moreover in this formalism one can see immediately whether two terms can give interference and also the complications due to spin are greatly reduced.

3. One-loop corrections

The radiative corrections to the lowest-order amplitude come from the self-energy of the gauge fields, the mixing term between the photon and the neutral vector boson, the self-energy of the electrons, the vertex parts and the box diagrams. Pictorially they are shown in fig. 2. The diagrams contributing to (a), (b) are shown in sect. 4 and in appendix C. The self-energy of the electron (diagrams (c), (d)) is discussed in sect. 5 and the diagrams are again in appendix C. The vertex parts are listed in sect. 6 and the box diagrams in sect. 7.

We have also to include the contribution coming from soft-photon bremsstrahlung (sect. 8) and the renormalization counter terms (sect. 9). From all these contributions we will derive the total differential cross section in the form

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^0}{d\Omega} + \frac{d\sigma^{(1)}}{d\Omega} + \frac{d\sigma^{\text{ren}}}{d\Omega}, \quad (3.1)$$

Fig. 2. One-loop corrections.

where $d\sigma^0/d\Omega$ is the lowest-order cross section given in sect. 2.

$$\frac{d\sigma^{(1)}}{d\Omega} = \frac{d\sigma^\Pi}{d\Omega} + \frac{d\sigma^\Sigma}{d\Omega} + \frac{d\sigma^V}{d\Omega} + \frac{d\sigma^{\text{box}}}{d\Omega} + \frac{d\sigma^{\text{cros}}}{d\Omega} + \frac{d\sigma^{\text{brem}}}{d\Omega} \quad (3.2)$$

is the sum of the interferences with the lowest-order amplitude of the various one-loop corrections plus the bremsstrahlung and $d\sigma^{\text{ren}}/d\Omega$ is the counter term coming from the renormalization of the weak mixing angle and the coupling constant.

4. Self-energy of the vector bosons

The diagrams (a) and (b) of fig. 2 correspond to the configurations of fig. 3. The diagrams contributing to the various blobs are listed in appendix C together with their analytical expressions. If we define as in eqs. (C.1), (C.2), (C.3) the following self-energies

$$\begin{aligned} & \text{Diagram 1: } \mu \text{---} \text{wavy line } A \text{---} \text{blob} \text{---} \text{wavy line } A \text{---} \nu, \quad \Pi_{\mu\nu}^{22}(k) = g^2 \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) F_{22}(k^2), \\ & \text{Diagram 2: } \mu \text{---} \text{wavy line } A \text{---} \text{blob} \text{---} \text{dashed line } W_0 \text{---} \nu, \quad \Pi_{\mu\nu}^{12}(k) = \Pi_{\mu\nu}^{21}(k) = g^2 (\delta_{\mu\nu} F_{12}(k^2) + k_\mu k_\nu A_{12}(k^2)), \\ & \text{Diagram 3: } \mu \text{---} \text{dashed line } W_0 \text{---} \text{blob} \text{---} \text{dashed line } W_0 \text{---} \nu, \quad \Pi_{\mu\nu}^{11}(k) = g^2 (\delta_{\mu\nu} F_{11}(k^2) + k_\mu k_\nu A_{11}(k^2)), \end{aligned}$$

we can derive as in ref. [11] the dressed propagators relative to the various transitions (terms proportional to $k_\mu k_\nu$ vanish or give contributions proportional to the

Fig. 3. The self-energy diagrams on the internal lines.

electron mass)

$$\begin{aligned}\Delta_{\mu\nu}^{11}(x) &= \frac{1}{(2\pi)^4 i} \frac{\delta_{\mu\nu}}{x + M_0^2 - g^2 \tilde{F}_{11}(x)}, \\ \Delta_{\mu\nu}^{12}(x) &= \frac{1}{(2\pi)^4 i} \frac{\delta_{\mu\nu} g^2 \tilde{F}_{12}(x)}{[x + M_0^2 - g^2 \tilde{F}_{11}(x)][x - g^2 \tilde{F}_{22}(x)]}, \\ \Delta_{\mu\nu}^{22}(x) &= \frac{1}{(2\pi)^4 i} \frac{\delta_{\mu\nu}}{x - g^2 \tilde{F}_{22}(x)},\end{aligned}\quad (4.1)$$

with

$$\tilde{F}_{ij}(x) = F_{ij}(x)/(2\pi)^4 i.$$

By separating the external currents in left- and right-handed parts and retaining only terms of order g^2 in the propagators it is easy to realize that the contribution to the cross section coming from the eight diagrams listed above can be put in the following form:

$$\begin{aligned}\frac{d\sigma^\Pi}{d\Omega} &= \left[\frac{g^2}{4\pi}\right]^3 \frac{1}{\pi} \frac{1}{2S} \{U^2[\Delta_{LL}(-S) + \Delta_{LL}(-T)][\Pi_{LL}(-S) + \Pi_{LL}(-T)] \\ &\quad + U^2[\Delta_{RR}(-S) + \Delta_{RR}(-T)][\Pi_{RR}(-S) + \Pi_{RR}(-T)] + 2T^2\Delta_{RL}(-S)\Pi_{RL}(-S) \\ &\quad + 2S^2\Delta_{RL}(-T)\Pi_{RL}(-T)\},\end{aligned}\quad (4.2)$$

where

$$\begin{aligned}\Pi_{LL}(x) &= 4\pi^2 \text{Real} \{s_w^2 \tilde{F}_{22}(x) D^2(x) + g_L^2 (\tilde{F}_{11}(x) - \tilde{F}_{11}(-M_0^2)) \Delta^2(x) \\ &\quad - 2s_w g_L \tilde{F}_{12}(x) D(x) \Delta(x)\}, \\ \Pi_{RL}(x) &= 4\pi^2 \text{Real} \{s_w \tilde{F}_{22}(x) D^2(x) + g_R g_L (\tilde{F}_{11}(x) - \tilde{F}_{11}(-M_0^2)) \Delta^2(x) \\ &\quad - s_w (g_L + g_R) \tilde{F}_{12}(x) D(x) \Delta(x)\}, \\ \Pi_{RR}(x) &= 4\pi^2 \text{Real} \{s_w^2 \tilde{F}_{22}(x) D^2(x) + g_R^2 (\tilde{F}_{11}(x) - \tilde{F}_{11}(-M_0^2)) \Delta^2(x) \\ &\quad - 2s_w g_R \tilde{F}_{12}(x) D(x) \Delta(x)\}.\end{aligned}\quad (4.3)$$

5. Self-energy of the electron

The three diagrams contributing to the electron self-energy are shown in appendix C(v). From their sum the general expression

$$\tilde{\Sigma}(k) = ik[A + B\gamma_5] + m_u C \quad (5.1)$$

is easily derived where A, B, C are functions of k^2 , of the internal masses and of the various coupling constants and m_u is the unrenormalized electron mass.

Assuming the decomposition (5.1), the dressed propagator is given by ($\tilde{\Sigma} = \Sigma/(2\pi)^4 i$)

$$S(k) = \frac{1}{(2\pi)^4 i} [ik + m_u - \tilde{\Sigma}(k)]^{-1}. \quad (5.2)$$

Taking the inverse of the matrix one gets

$$S = \frac{1}{(2\pi)^4 i} \frac{-ik[1 - A - B\gamma_5] + m_u[1 - C]}{k^2[1 - A]^2 + m_u^2[1 - C]^2},$$

from which, by requiring a pole in $k^2 = -m^2$, it follows

$$-m^2[1 - \underline{A}]^2 + m_u^2[1 - \underline{C}]^2 = 0$$

(underlined quantities are calculated in $k^2 = -m^2$), and since A , B and C are of order g^2

$$m = m_u + m_u[A - \underline{C}].$$

Substituting everywhere m_u in terms of m , expanding around $k^2 = -m^2$ and retaining only linear terms in $(k^2 + m^2)$ (since we are interested in mass-shell contributions of the electron self-energy) we get

$$S = \frac{1}{(2\pi)^4 i} \frac{[-ik + m]Z + ik\underline{B}\gamma_5}{k^2 + m^2}, \quad (5.3)$$

where

$$Z = 1 + A + 2m^2[C' - A'],$$

$$C' = \left. \frac{\partial C}{\partial k^2} \right|_{k^2 = -m^2}, \quad A' = \left. \frac{\partial A}{\partial k^2} \right|_{k^2 = -m^2}.$$

Introducing $Z_L = Z - \underline{B}$ and $Z_R = Z + \underline{B}$ one has for the residual matrix in the pole

$$(2\pi)^4 i \lim_{k^2 \rightarrow -m^2} (k^2 + m^2) S(k) = -ik \frac{1 + \gamma_5}{2} Z_L - ik \frac{1 - \gamma_5}{2} Z_R + mZ,$$

which requires the following counterterms,

$$-\bar{\psi}_L \not{\partial} \psi_L [Z_L - 1] - \bar{\psi}_R \not{\partial} \psi_R [Z_R - 1] - m[Z - 1][\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L],$$

to be added to the free Lagrangian [7] for the electron field

$$-\bar{\psi}_L \not{\partial} \psi_L - \bar{\psi}_R \not{\partial} \psi_R - m[\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L].$$

The renormalized Lagrangian then looks like

$$-\bar{\psi}_L \not{\partial} \psi_L Z_L - \bar{\psi}_R \not{\partial} \psi_R Z_R - mZ[\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L].$$

As one could have expected the left- and right-handed fields renormalize in two

different ways. Defining the renormalized fields ψ_L^0 and ψ_R^0 by

$$\psi_L^0 = \sqrt{Z_L} \psi_L, \quad \psi_R^0 = \sqrt{Z_R} \psi_R,$$

it turns out also that the mass term is exactly

$$-m[\bar{\psi}_L^0 \psi_R^0 + \bar{\psi}_R^0 \psi_L^0],$$

since $\sqrt{Z_L Z_R} = Z$ to order g^2 .

The contribution to the amplitude coming from external lines self-energy is then derived in the usual way from the reduction formula for S -matrix elements. For example for an outgoing electron the relevant part is

$$\frac{1}{2m} \left[\frac{\bar{u}_L}{\sqrt{Z_L}} + \frac{\bar{u}_R}{\sqrt{Z_R}} \right] [-ik(Z - \underline{B}\gamma_5) + mZ] \dots, \quad (5.4)$$

with

$$u_L = \frac{1}{2}(1 + \gamma_5)u, \quad u_R = \frac{1}{2}(1 - \gamma_5)u.$$

Carrying out the products and using the equations of motions we can get the contribution due to the electron self-energy by replacing $\bar{u}$ with

$$\frac{1}{2}\bar{u}(Z - 1 - \underline{B}\gamma_5). \quad (5.5)$$

Extracting as usual the left- and right-handed parts of the amplitude and defining

$$\begin{aligned} g^2 \Sigma_{LL} &= 4\pi^2 \text{Real} \{2(Z - 1 - \underline{B})\}, \\ g^2 \Sigma_{RL} &= 4\pi^2 \text{Real} \{2(Z - 1)\}, \\ g^2 \Sigma_{RR} &= 4\pi^2 \text{Real} \{2(Z - 1 + \underline{B})\}, \end{aligned} \quad (5.6)$$

the contribution to the cross section is

$$\begin{aligned} \frac{d\sigma^\Sigma}{d\Omega} &= \left[\frac{g^2}{4\pi} \right]^3 \frac{1}{\pi} \frac{1}{2S} \{ U^2 [\Delta_{LL}(-S) + \Delta_{LL}(-T)]^2 \Sigma_{LL} \\ &\quad + U^2 [\Delta_{RR}(-S) + \Delta_{RR}(-T)]^2 \Sigma_{RR} \\ &\quad + [2T^2 \Delta_{RL}^2(-S) + 2S^2 \Delta_{RL}^2(-T)] \Sigma_{RL} \}. \end{aligned} \quad (5.7)$$

The infrared divergent part contained in $d\sigma^\Sigma/d\Omega$ factorizes into $d\sigma^0/d\Omega$ as follows. First we note that

$$\Sigma_{LL}^{\text{IR}} = \Sigma_{RR}^{\text{IR}} = \Sigma_{RL}^{\text{IR}} = \Sigma^{\text{IR}};$$

then the expression for Σ^{IR} can be derived from appendix C(v) in the form

$$\Sigma^{\text{IR}} = s_w^2 \ln \frac{m^2}{\lambda^2},$$

so that

$$\left(\frac{d\sigma^\Sigma}{d\Omega}\right)_{\text{IR}} = \frac{d\sigma^0}{d\Omega} \frac{2\alpha}{\pi} \ln \frac{m^2}{\lambda^2}. \quad (5.8)$$

6. Vertex corrections

Seven diagrams contribute to the total amplitude, they are shown in fig. 4. Vertices (a)–(e) have a very similar structure and can be treated together. Indeed let us consider the general diagram of fig. 5. The upper external dashed line can be a photon or a neutral vector boson, the internal dashed line with mass M_1 can be again a photon or a W_0 (in this case the two internal fermions are electrons) or a charged vector boson (in this case we have two internal neutrinos). General V–A couplings are assumed in the vertices and the expression for the diagram is the following (M_2 and M_3 are lepton masses and we neglect them systematically in the numerator):

$$V = (ig)^3 \bar{u}_2 \gamma_\lambda (V_1 + A_1 \gamma_5) \int d^n q \frac{[-i(q - P_2)] \gamma_\mu (V_2 + A_2 \gamma_5) [-i(q + P_1)]}{(1) (2) (3)} \times \gamma_\lambda (V_3 + A_3 \gamma_5) u_1, \quad (6.1)$$

Fig. 4. The vertex corrections.

Fig. 5. General diagram for the vertices (a)–(e).

where

$$(1) = q^2 + M_1^2, \quad (2) = (q + P_1)^2 + M_2^2, \quad (3) = (q - P_2)^2 + M_3^2.$$

$\bar{u}_2$ is referred to the outgoing electron (or incoming positron) with momentum P_2 while u_1 is the incoming electron (or outgoing positron) with momentum P_1 relatively to the four different positions in which the vertex has to be inserted.

Introducing the notation

$$V + A\gamma_5 = (V_1 + A_1\gamma_5)(V_2 + A_2\gamma_5)(V_3 + A_3\gamma_5),$$

and carrying out some products, we get from appendix B the reduced expression

$$V = (ig)^3 \bar{u}_2 \gamma_\tau \gamma_\mu \gamma_\nu (V + A\gamma_5) u_1 \{ (n-2) C_{\nu\tau} - 2(P_1^\tau P_2^\nu C_0 - P_1^\tau C_\nu + P_2^\nu C_\tau) \}. \quad (6.2)$$

The term $(n-2)$ comes from the reduction formulas of γ -matrices for dimensional regularization since only $C_{\nu\tau}$ contains ultraviolet divergences. Introducing the form factors of the appendix, using the equations of motion for massless external spinors (e.g., $\not{P}_1 u_1 = \bar{u}_2 \not{P}_2 = 0$) and substituting $n-2 = (n-4) + 2$ we get at the end

$$V = (ig)^3 \bar{u}_2 \gamma_\mu (V + A\gamma_5) u_1 \{ 4(P_1 \cdot P_2) [C_0 + C_{11} - C_{22} + C_{23}] - 4C_{24} + 2i\pi^2 \} \quad (6.3)$$

(the term $i\pi^2$ comes from retaining the finite contribution arising from the residue of the pole in $n=4$ which appears in C_{24} only).

Now we have to specify for every diagram the internal masses as arguments of the C -functions together with the corresponding coupling constants in the vertices (we will give as usual the chiral combinations $V+A$ and $V-A$):

$$\text{Vertex (a): } M_1 = \lambda, \quad M_2 = m, \quad M_3 = m,$$

$$V + A = -s_w^3, \quad V - A = -s_w^3;$$

Vertex (b): $M_1 = \lambda$, $M_2 = m$, $M_3 = m$,

$$V + A = s_w^2 g_L, \quad V - A = s_w^2 g_R;$$

Vertex (c): $M_1 = M_0$, $M_2 = m$, $M_3 = m$,

$$V + A = -s_w g_L^2, \quad V - A = -s_w g_R^2;$$

Vertex (d): $M_1 = M_0$, $M_2 = m$, $M_3 = m$,

$$V + A = g_L^3, \quad V - A = g_R^3;$$

Vertex (e): $M_1 = M$, $M_2 = 0$, $M_3 = 0$,

$$V + A = \frac{1}{4c_w}, \quad V - A = 0$$

(a fictitious photon mass has been introduced to regularize the infrared divergence appearing in vertices (a) and (b)).

The vertices (f) and (g) can be treated in a similar way, moreover the internal masses are the same in both cases. The general structure is shown in fig. 6. Let us introduce

$$U_{\alpha\mu\beta} \equiv -\delta_{\alpha\mu}(2P_1 + P_2 + q)_\beta + \delta_{\mu\beta}(2P_2 + P_1 - q)_\alpha + \delta_{\alpha\beta}(P_1 - P_2 + 2q)_\mu. \quad (6.4)$$

The vertex has then the following form

$$U = (-s_w, -c_w)^{\frac{1}{4}} (ig)^3 \bar{u}_2 \gamma_\beta \gamma_\delta \gamma_\alpha (1 + \gamma_5) u_1 \int d^n q \frac{q_\delta U_{\alpha\mu\beta}}{(1)(2)(3)}, \quad (6.5)$$

where

$$(1) = q^2, \quad (2) = (q + P_1)^2 + M^2, \quad (3) = (q - P_2)^2 + M^2,$$

Fig. 6. General diagram for vertices (f) and (g).

and the bracket $(-s_w, -c_w)$ stands for the two possibilities of having an external photon or neutral vector boson.

Introducing as usual the form factors and using the equations of motion we get after some algebra

$$U = (-s_w, -c_w) \frac{1}{2} (ig)^3 \bar{u}_2 \gamma_\mu (1 + \gamma_5) u_1 [2(P_1 \cdot P_2)(C_{11} - C_{22} + C_{23}) - 6C_{24} + i\pi^2] : \quad (6.6)$$

again the $i\pi^2$ comes from a term

$$(2 - 2n)C_{24} = -2[(n - 4) + 3]C_{24}.$$

To give an expression of the contribution to the cross section coming from all vertex corrections we have to introduce some short notations. We note that the two different combinations of C -functions depend only on the internal masses and on $x = (P_1 + P_2)^2$, since the two external electrons are on-mass-shell.

Let us define the combination of form factors of vertices (a)–(e) ($x \sim 2P_1 \cdot P_2$)

$$2x[C_0 + C_{11} - C_{22} + C_{23}] - 4C_{24} + 2i\pi^2 = \begin{cases} V_G(x) & \text{diagrams (a), (b),} \\ V_Z(x) & \text{diagrams (c), (d),} \\ V_w(x) & \text{diagram (e),} \end{cases} \quad (6.7)$$

and also the combination of vertices (f) and (g)

$$x[C_{11} - C_{22} + C_{23}] - 6C_{24} + i\pi^2 = U_w(x). \quad (6.8)$$

With these definitions it is possible to put the contribution to the cross section in the form

$$\begin{aligned} \frac{d\sigma^V}{d\Omega} = & \left[\frac{g^2}{4\pi} \right]^3 \frac{1}{\pi} \frac{1}{2S} [U^2[\Delta_{LL}(-S) + \Delta_{LL}(-T)][V_{LL}(-S) + V_{LL}(-T)] \\ & + U^2[\Delta_{RR}(-S) + \Delta_{RR}(-T)][V_{RR}(-S) + V_{RR}(-T)] \\ & + 2T^2\Delta_{RL}(-S)V_{RL}(-S) + 2S^2\Delta_{RL}(-T)V_{RL}(-T)], \end{aligned} \quad (6.9)$$

with

$$\begin{aligned} V_{LL}(x) = & -\frac{1}{2\pi^2} \text{Im} \{ (s_w^2 V_G(x) + g_L^2 V_Z(x)) \Delta_{LL}(x) + \frac{g_L}{4c_w} \Delta(x) V_w(x) \\ & + (s_w^2 D(x) - c_w g_L \Delta(x)) U_w(x) \}, \\ V_{RL}(x) = & -\frac{1}{4\pi^2} \text{Im} \left\{ (2s_w^2 V_G(x) + (g_L^2 + g_R^2) V_Z(x)) \Delta_{RL}(x) + \frac{g_R}{4c_w} \Delta(x) V_w(x) \right. \\ & \left. + (s_w^2 D(x) - c_w g_R \Delta(x)) U_w(x) \right\}, \\ V_{RR}(x) = & -\frac{1}{2\pi^2} \text{Im} \{ (s_w^2 V_G(x) + g_R^2 V_Z(x)) \Delta_{RR}(x) \}, \end{aligned} \quad (6.10)$$

and the imaginary part is taken due to the interference with the lowest-order amplitude which contains the factor $(2\pi)^4 i$.

The function $V_G(x)$ is infrared divergent since it contains the 3-point function $C_0(x, \lambda, m, m)$ (see appendix B). The infrared divergence associated to C_0 is known from QED, we will simply give the result. Defining C_0^{IR} the divergent part, we have

$$C_0^{\text{IR}}(x, \lambda, m, m) = \frac{1}{2} i \pi^2 \ln \frac{m^2}{\lambda^2} \theta(-x), \quad (6.11)$$

and the function θ is defined in eq. (D.3).

Since we shall use the high-energy limit of the function $\theta(x)$ it is worthwhile to give this expression in the case of $x = -S, -T, -U$. We get

$$\begin{aligned} \theta(S) &\sim -\frac{2}{S} \ln \frac{S}{m^2} \equiv -\frac{4}{S} v', \\ \theta(T) &\sim -\frac{2}{T} \ln \frac{S}{m^2} \frac{(1 - \cos \theta)}{2} \equiv -\frac{4}{T} w', \\ \theta(U) &\sim -\frac{2}{U} \ln \frac{S}{m^2} \frac{(1 + \cos \theta)}{2} \equiv -\frac{4}{U} u'. \end{aligned} \quad (6.12)$$

If we define the auxiliary function

$$f_\lambda(-x) = \frac{1}{2} x \ln \frac{m^2}{\lambda^2} \theta(-x), \quad (6.13)$$

and in particular

$$f_\lambda(S) = 2v' \ln \frac{m^2}{\lambda^2}, \quad f_\lambda(T) = 2w' \ln \frac{m^2}{\lambda^2}, \quad f_\lambda(U) = 2u' \ln \frac{m^2}{\lambda^2}, \quad (6.14)$$

we get

$$C_0^{\text{IR}}(x, \lambda, m, m) = \frac{i\pi^2}{x} f_\lambda(-x), \quad (6.15)$$

from which we can derive

$$V_G^{\text{IR}}(x) = 2i\pi^2 f_\lambda(-x), \quad (6.16)$$

and we can easily calculate the infrared divergent part coming from the vertex corrections.

Since

$$\begin{aligned} V_{\text{LL}}^{\text{IR}}(x) &= -s_w^2 f_\lambda(-x) \Delta_{\text{LL}}(x), \\ V_{\text{RL}}^{\text{IR}}(x) &= -s_w^2 f_\lambda(-x) \Delta_{\text{RL}}(x), \\ V_{\text{RR}}^{\text{IR}}(x) &= -s_w^2 f_\lambda(-x) \Delta_{\text{RR}}(x), \end{aligned} \quad (6.17)$$

we get in the end

$$\begin{aligned}
 \left(\frac{d\sigma^V}{d\Omega} \right)_{\text{IR}} = & - \left[\frac{g^2}{4\pi} \right]^2 \frac{\alpha}{\pi} \frac{1}{2S} \{ U^2 [\Delta_{\text{LL}}(-S) + \Delta_{\text{LL}}(-T)] \\
 & \times [f_\lambda(S)\Delta_{\text{LL}}(-S) + f_\lambda(T)\Delta_{\text{LL}}(-T)] \\
 & + U^2 [\Delta_{\text{RR}}(-S) + \Delta_{\text{RR}}(-T)] [f_\lambda(S)\Delta_{\text{RR}}(-S) + f_\lambda(T)\Delta_{\text{RR}}(-T)] \\
 & + 2T^2 \Delta_{\text{RL}}^2(-S) f_\lambda(S) + 2S^2 \Delta_{\text{RL}}^2(-T) f_\lambda(T) \}. \quad (6.18)
 \end{aligned}$$

7. Box diagrams

There are nine different box diagrams which can give interference with the lowest-order amplitude to yield the total cross section. They are shown in fig. 7. The external lines can correspond to the S - or T -channel so that we get a total number of eighteen diagrams. Following the same procedure we have used for the vertex corrections we shall present a reduced expression for the amplitude in the general case from which the contributions of the various diagrams to the cross section can be derived.

A (direct) box diagram with general $V-A$ couplings and arbitrary internal masses has the pictorial structure shown in fig. 8. M_1 and M_3 are gauge fields masses while M_2 and M_4 are lepton masses (electrons or neutrinos). Moreover in

Fig. 7. The direct and crossed box diagrams.

Fig. 8. A general box diagram.

our case $M_2 = M_4$ and $V_1 = V_4$, $A_1 = A_4$, $V_2 = V_3$, $A_2 = A_3$. In this way, defining

$$V + A\gamma_5 = (V_1 + A_1\gamma_5)(V_2 + A_2\gamma_5),$$

the couplings on the two spinorial strings are the same. The expression for the diagram is the following

$$M^{\text{box}} = g^4 \bar{u}_2 \gamma_\nu (V_2 + A_2 \gamma_5) \int d^4 q \times \frac{[-i(\not{q} + \not{P}_1)] \gamma_\mu (V_1 + A_1 \gamma_5) u_1 \bar{u}_4 \gamma_\mu (V_1 + A_1 \gamma_5) [-i(\not{q} - \not{P}_4)]}{(1) (2) (3) (4)} \times \gamma_\nu (V_2 + A_2 \gamma_5) u_3, \quad (7.1)$$

where

$$(1) = q^2 + M_1^2, \quad (2) = (q + P_1)^2 + M_2^2, \\ (3) = (q + P_1 + P_2)^2 + M_3^2, \quad (4) = (q - P_4)^2 + M_2^2.$$

From appendix B we get

$$M^{\text{box}} = -g^4 \bar{u}_2 \gamma_\nu \gamma_\lambda \gamma_\mu (V + A\gamma_5) u_1 \bar{u}_4 \gamma_\mu \gamma_\rho \gamma_\nu (V + A\gamma_5) u_3 \times [D_{\lambda\rho} - P_4^\rho D_\lambda + P_1^\lambda D_\rho - P_1^\lambda P_4^\rho D_0]. \quad (7.2)$$

Introducing as usual the form factors and contracting the external momenta with

their own spinors we get (see (B.11)) the reduced expression ($x = (P_1 + P_2)^2$)

$$M^{\text{box}} = -g^4 \{ (V^2 - A^2) [J_{\text{LR}}(x) + J_{\text{RL}}(x)] [4F_4 - (P_1 \cdot P_2)F_1 - (P_2 \cdot P_3)F_2] \\ + [(V + A)^2 J_{1\mu}^{\text{L}}(x) J_{2\nu}^{\text{L}}(x) + (V - A)^2 J_{1\mu}^{\text{R}}(x) J_{2\nu}^{\text{R}}(x)] [\delta_{\mu\nu} F_4 - P_3^\mu P_2^\nu F_3] \} \quad (7.3)$$

($J_{1\mu}^{\text{L,R}}$ is referred to (P_1, P_2) while $J_{2\nu}^{\text{L,R}}$ to (P_3, P_4) , see (2.6)). Since the external momenta are always on the electron mass-shell the F -functions depend only on the internal masses and on two squared momenta for which we can choose $x = (P_1 + P_2)^2$ and $y = (P_2 + P_3)^2$. For the S -channel direct box diagrams we have $x = -S$ and $y = -T$, while for the T -channel, $x = -T$ and $y = -S$. We now have to specify the internal masses and the coupling constants for the various diagrams of fig. 7. For diagram (a)

$$M_1 = \lambda, \quad M_2 = m, \quad M_3 = \lambda, \quad M_4 = m, \\ V^2 - A^2 = s_{\text{w}}^4, \quad (V + A)^2 = s_{\text{w}}^4, \quad (V - A)^2 = s_{\text{w}}^4.$$

Diagrams (b) and (c) are equal (the form factors D_{ij} change from (b) to (c), but D_0 and the combinations F_1, F_2, F_3, F_4 are the same) so that we give only one choice of the internal masses

$$M_1 = \lambda, \quad M_2 = m, \quad M_3 = M_0, \quad M_4 = m, \\ V^2 - A^2 = s_{\text{w}}^2 g_{\text{R}} g_{\text{L}}, \quad (V + A)^2 = s_{\text{w}}^2 g_{\text{L}}^2, \quad (V - A)^2 = s_{\text{w}}^2 g_{\text{R}}^2.$$

For diagram (d)

$$M_1 = M_0, \quad M_2 = m, \quad M_3 = M_0, \quad M_4 = m, \\ V^2 - A^2 = g_{\text{R}}^2 g_{\text{L}}^2, \quad (V + A)^2 = g_{\text{L}}^4, \quad (V - A)^2 = g_{\text{R}}^4,$$

and for diagram (e)

$$M_1 = M, \quad M_2 = 0, \quad M_3 = M, \quad M_4 = 0, \\ V^2 - A^2 = 0, \quad (V + A)^2 = \frac{1}{4}, \quad (V - A)^2 = 0$$

(again λ is the photon mass introduced to regularize the infrared divergence appearing in diagrams (a), (b) and (c)).

Analogous calculations can be carried out in the case of the crossed box diagrams (f)–(i) of fig. 7. Using the same notations one finds

$$M^{\text{cross}} = g^4 \{ [(V + A)^2 J_{\text{LL}}(x) + (V - A)^2 J_{\text{RR}}(x)] [4F_4 - (P_1 \cdot P_2)F_1 - (P_2 \cdot P_3)F_2] \\ + (V^2 - A^2) [J_{1\mu}^{\text{L}}(x) J_{2\nu}^{\text{R}}(x) + J_{1\mu}^{\text{R}}(x) J_{2\nu}^{\text{L}}(x)] [\delta_{\mu\nu} F_4 - P_3^\mu P_2^\nu F_3] \}, \quad (7.4)$$

so that we see that there is an overall minus sign and an interchange of the spinorial currents relatively to their left-right character apart from the dependence on the external momenta. Indeed for the S -channel crossed diagrams $x = -S$ and $y = -U$, while in the T -channel, $x = -T$ and $y = -U$.

Again we have to introduce some short notation to give the expression of the contribution to the cross section coming from the box diagrams. We define the quantity

$$4F_4(x, y) - \frac{1}{2}xF_1(x, y) - \frac{1}{2}yF_2(x, y), \quad (7.5)$$

to be given for the various diagrams by the following notations:

Diagrams	Notations	
(a), (f),	$F_{GG}(x, y),$	(7.5a)

(b), (c), (g), (h),	$F_{GZ}(x, y),$	(7.5b)
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(d), (i),	$F_{ZZ}(x, y).$	(7.5c)
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Analogously the quantity

$$F_4(x, y) - \frac{1}{2}yF_3(x, y) \quad (7.6)$$

can be defined as indicated below

Diagrams	Notations	
(a), (f),	$H_{GG}(x, y),$	(7.6a)

(b), (c), (g), (h),	$H_{GZ}(x, y),$	(7.6b)
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(d), (i),	$H_{ZZ}(x, y),$	(7.6c)
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(e),	$H_{ww}(x, y).$	(7.6d)
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The ten direct box diagrams give then the following contribution to the cross section:

$$\begin{aligned} \frac{d\sigma^{\text{box}}}{d\Omega} = & -\left[\frac{g^2}{4\pi}\right]^3 \frac{1}{\pi} \frac{1}{2S} \{U^2[\Delta_{LL}(-S) + \Delta_{LL}(-T)][H_{LL}(-S, -T) + H_{LL}(-T, -S)] \\ & + U^2[\Delta_{RR}(-S) + \Delta_{RR}(-T)][H_{RR}(-S, -T) + H_{RR}(-T, -S)] \\ & + 2T^2\Delta_{RL}(-S)F_{RL}(-S, -T) + 2S^2\Delta_{RL}(-T)F_{RL}(-T, -S)\}, \end{aligned} \quad (7.7)$$

while the eight crossed diagrams give

$$\begin{aligned} \frac{d\sigma^{\text{cross}}}{d\Omega} = & \left[\frac{g^2}{4\pi}\right]^3 \frac{1}{\pi} \frac{1}{2S} \{U^2[\Delta_{LL}(-S) + \Delta_{LL}(-T)][F_{LL}(-S, -U) + F_{LL}(-T, -U)] \\ & + U^2[\Delta_{RR}(-S) + \Delta_{RR}(-T)][F_{RR}(-S, -U) + F_{RR}(-T, -U)] \\ & + 2T^2\Delta_{RL}(-S)H_{RL}(-S, -U) + 2S^2\Delta_{RL}(-T)H_{RL}(-T, U)\}, \end{aligned} \quad (7.8)$$

where we have introduced

$$\begin{aligned}
 F_{LL}(x, y) &= -\frac{1}{\pi^2} \text{Im} \{s_w^4 F_{GG}(x, y) + 2s_w^2 g_L^2 F_{GZ}(x, y) + g_L^4 F_{ZZ}(x, y)\}, \\
 F_{RL}(x, y) &= -\frac{1}{\pi^2} \text{Im} \{s_w^4 F_{GG}(x, y) + 2s_w^2 g_R g_L F_{GZ}(x, y) + g_R^2 g_L^2 F_{ZZ}(x, y)\}, \\
 F_{RR}(x, y) &= -\frac{1}{\pi^2} \text{Im} \{s_w^4 F_{GG}(x, y) + 2s_w^2 g_R^2 F_{GZ}(x, y) + g_R^4 F_{ZZ}(x, y)\}; \quad (7.9) \\
 H_{LL}(x, y) &= -\frac{1}{\pi^2} \text{Im} \{s_w^4 H_{GG}(x, y) + 2s_w^2 g_L^2 H_{GZ}(x, y) \\
 &\quad + g_L^4 H_{ZZ}(x, y) + \frac{1}{4} H_{ww}(x, y)\}, \\
 H_{RL}(x, y) &= -\frac{1}{\pi^2} \text{Im} \{s_w^4 H_{GG}(x, y) + 2s_w^2 g_R g_L H_{GZ}(x, y) + g_R^2 g_L^2 H_{ZZ}(x, y)\}, \\
 H_{RR}(x, y) &= -\frac{1}{\pi^2} \text{Im} \{s_w^4 H_{GG}(x, y) + 2s_w^2 g_R^2 H_{GZ}(x, y) + g_R^4 H_{ZZ}(x, y)\}. \quad (7.10)
 \end{aligned}$$

We note that in the case of pure QED in the S -channel (as in the case of $e^+e^- \rightarrow \mu^+\mu^-$) the two cross sections reduce to

$$\begin{aligned}
 \frac{d\sigma^{\text{box}}}{d\Omega} &= -\frac{\alpha^3}{\pi} \frac{1}{2S} [2U^2 D(-S)H(-S, -T) + 2T^2 D(-S)F(-S, -T)], \\
 \frac{d\sigma^{\text{cross}}}{d\Omega} &= \frac{\alpha^3}{\pi} \frac{1}{2S} [2U^2 D(-S)F(-S, -U) + 2T^2 D(-S)H(-S, -U)], \quad (7.11)
 \end{aligned}$$

where now

$$H(x, y) = -\frac{1}{\pi^2} \text{Im} H_{GG}(x, y), \quad F(x, y) = -\frac{1}{\pi^2} \text{Im} F_{GG}(x, y).$$

In this case we see that $d\sigma^{\text{cross}}/d\Omega$ is obtained from $d\sigma^{\text{box}}/d\Omega$ by adding a minus sign and by the overall replacement $T \leftrightarrow U$. To finish our discussion of the box diagrams we give a brief description of the infrared behaviour of our form factors. First we want to stress that with our choice of the internal masses only the functions F_2 and F_3 defined in eq. (B.11) contain infrared divergences; moreover their divergent parts are the same. Indeed by direct investigation of the diagrams (a) and (f) of fig. 7 we see that the infrared divergences are associated to the points of q -space, $q = 0$ and $q = -(P_1 + P_2)$ (see fig. 8): then, by means of the simple rule described in ref. [12], we can extract the divergent part of the vector and tensor 4-point functions D_μ and $D_{\mu\nu}$ by calculating the residue of the singularity, i.e., the point in which three denominators vanish simultaneously. In the case of diagrams

(a) and (f) of fig. 7 we get

$$\begin{aligned} D_\mu^{\text{IR}} &= -\frac{1}{2}(P_1^\mu + P_2^\mu)D_0^{\text{IR}}, \\ D_{\mu\nu}^{\text{IR}} &= \frac{1}{2}(P_1^\mu + P_2^\mu)(P_1^\nu + P_2^\nu)D_0^{\text{IR}} \end{aligned} \quad (7.12)$$

(the factor $\frac{1}{2}$ is present because the divergent part of D_0 is obtained by adding the two equal residues in $q=0$ and $q=-(P_1+P_2)$). Therefore we see that

$$\begin{aligned} D_{11}^{\text{IR}} &= D_{12}^{\text{IR}} = -\frac{1}{2}D_0^{\text{IR}}, \\ D_{24}^{\text{IR}} &= \frac{1}{2}D_0^{\text{IR}}. \end{aligned} \quad (7.13)$$

In QED the divergent part of D_0 is known from ref. [3], and we only give the result:

$$D_0^{\text{IR}}(x, y, \lambda, m, \lambda, m) = 2D(x)C_0^{\text{IR}}(y, \lambda, m, m)$$

($D(x)$ is defined in (2.2)) so that from (6.15) we get

$$D_0^{\text{IR}}(x, y, \lambda, m, \lambda, m) = \frac{2i\pi^2}{y} D(x) f_\lambda(-y). \quad (7.14)$$

The box diagrams (b) and (g) of fig. 7 have a singularity in $q=-(P_1+P_2)$ so that again only the scalar 4-point function D_0 and the form-factors D_{11} , D_{12} and D_{24} diverge. We get in this case

$$\begin{aligned} D_{11}^{\text{IR}} &= D_{12}^{\text{IR}} = -D_0^{\text{IR}}, \\ D_{24}^{\text{IR}} &= D_0^{\text{IR}}, \end{aligned} \quad (7.15)$$

while in the case of the box diagrams (c) and (h) of fig. 7 only D_0 is divergent since the only singularity is in $q=0$. The infrared divergent part of D_0 in the case of the box diagrams (b), (c), (g) and (h) can then be easily calculated by introducing as in ref. [3] the following set of momenta

$$P = \frac{1}{2}(P_1 + P_2), \quad Q = \frac{1}{2}(P_2 - P_1), \quad \Delta = \frac{1}{2}(P_4 - P_3),$$

which satisfy $P \cdot Q = P \cdot \Delta = 0$.

The scalar 4-point function with a photon and a neutral vector boson has the form

$$D_0 = \int d^4q \frac{1}{[(q-P)^2 + \lambda^2][Q][(q+P)^2 + M_0^2][\Delta]}$$

(with $[Q] = (q-Q)^2 + m^2$ and $[\Delta] = (q-\Delta)^2 + m^2$), when the fictitious photon mass is taken as M_1 (diagrams (c) and (h)) and has the same expression with $P \leftrightarrow -P$ when M_0 is chosen as M_1 (diagrams (b) and (g)). Since D_0 depends on P only through P^2 , the two integrals are the same. If we introduce the W_0 propagator

$1/(4P^2 + M_0^2)(4P^2 = -S, -T)$ we get

$$D_0 = \frac{1}{4P^2 + M_0^2} \int d^4q \frac{(P-q)^2 + 2(P^2 - q^2) + (P+q)^2 + M_0^2}{[[q-P]^2 + \lambda^2][\Delta][(q+P)^2 + M_0^2][\Delta]}.$$

Again the only divergent part is contained in the 3-point function so that

$$D_0^{\text{IR}}(x, y, \lambda, m, M_0, m) = \Delta(x) C_0^{\text{IR}}(y, \lambda, m, m),$$

and from (6.15) we get

$$D_0^{\text{IR}}(x, y, \lambda, m, M_0, m) = \frac{i\pi^2}{y} \Delta(x) f_\lambda(-y). \quad (7.16)$$

From (7.13)–(7.16) with the definitions (7.5) and (7.6) we get

$$\begin{aligned} H_{\text{GG}}^{\text{IR}}(x, y) &= F_{\text{GG}}^{\text{IR}}(x, y) = -i\pi^2 D(x) f_\lambda(-y), \\ H_{\text{GZ}}^{\text{IR}}(x, y) &= F_{\text{GZ}}^{\text{IR}}(x, y) = -\frac{1}{2}i\pi^2 \Delta(x) f_\lambda(-y), \end{aligned} \quad (7.17)$$

and from (7.10) we can derive

$$\begin{aligned} H_{\text{LL}}^{\text{IR}}(x, y) &= F_{\text{LL}}^{\text{IR}}(x, y) = s_w^2 \Delta_{\text{LL}}(x) f_\lambda(-y), \\ H_{\text{RL}}^{\text{IR}}(x, y) &= F_{\text{RL}}^{\text{IR}}(x, y) = s_w^2 \Delta_{\text{RL}}(x) f_\lambda(-y), \\ H_{\text{RR}}^{\text{IR}}(x, y) &= F_{\text{RR}}^{\text{IR}}(x, y) = s_w^2 \Delta_{\text{RR}}(x) f_\lambda(-y). \end{aligned} \quad (7.18)$$

From (7.7) and (7.8) we get

$$\begin{aligned} \left(\frac{d\sigma^{\text{box}}}{d\Omega} \right)_{\text{IR}} &= - \left[\frac{g^2}{4\pi} \right]^2 \frac{1}{\pi} \frac{1}{2S} \{ U^2 [\Delta_{\text{LL}}(-S) + \Delta_{\text{LL}}(-T)] \\ &\quad \times [f_\lambda(T) \Delta_{\text{LL}}(-S) + f_\lambda(S) \Delta_{\text{LL}}(-T)] \\ &\quad + U^2 [\Delta_{\text{RR}}(-S) + \Delta_{\text{RR}}(-T)] [f_\lambda(T) \Delta_{\text{RR}}(-S) + f_\lambda(S) \Delta_{\text{RR}}(-T)] \\ &\quad + 2T^2 \Delta_{\text{RL}}^2(-S) f_\lambda(T) + 2S^2 \Delta_{\text{RL}}^2(-T) f_\lambda(S) \}, \end{aligned} \quad (7.19)$$

$$\left(\frac{d\sigma^{\text{cros}}}{d\Omega} \right)_{\text{IR}} = \frac{d\sigma^0}{d\Omega} \frac{2\alpha}{\pi} f_\lambda(U). \quad (7.20)$$

Putting together (5.8), (6.18), (7.19) and (7.20), and recalling the definitions (6.14) we get the following expression for the infrared divergent part coming from all one-loop corrections:

$$\begin{aligned} &\left(\frac{d\sigma^\Sigma}{d\Omega} \right)_{\text{IR}} + \left(\frac{d\sigma^V}{d\Omega} \right)_{\text{IR}} + \left(\frac{d\sigma^{\text{box}}}{d\Omega} \right)_{\text{IR}} + \left(\frac{d\sigma^{\text{cros}}}{d\Omega} \right)_{\text{IR}} \\ &= \frac{d\sigma^0}{d\Omega} \frac{\alpha}{\pi} \left\{ 2(1 - 2v' + 2u' - 2w') \ln \frac{m^2}{\lambda^2} \right\}. \end{aligned} \quad (7.21)$$

8. Soft photon bremsstrahlung

The infrared divergences appearing in the calculation of one-loop diagrams are quite similar to the ones occurring in QED, therefore the cancellation of the divergent pieces against analogous terms coming from soft photon bremsstrahlung works in the usual way.

The sixteen diagrams pictorially represented in fig. 9 give in the soft photon approximation the well-known [12] contribution to the cross section

$$\frac{d\sigma^{\text{brem}}}{d\Omega} = \frac{d\sigma^0}{d\Omega} \frac{\alpha}{2\pi^2} f(S, T, U), \quad (8.1)$$

with

$$f(S, T, U) = \int \frac{d^3q}{2q_0} \left| \frac{K_1^\mu}{K_1 \cdot q} - \frac{Q_1^\mu}{Q_1 \cdot q} + \frac{Q_2^\mu}{Q_2 \cdot q} - \frac{K_2^\mu}{K_2 \cdot q} \right|^2, \quad (8.2)$$

and the integration over the soft photon momentum is carried out up to a maximum energy q_m and it is isotropically extended over the whole solid angle. The high-energy limit of the function $f(S, T, U)$ for Bhabha scattering can be found in ref. [4], we will simply quote the result:

$$f(S, T, U) = 2\pi \left[2(1 - 2v' + 2u' - 2w') \ln \frac{\lambda^2}{4q_m^2} + 2\text{SP}\left(\frac{1 + \cos \theta}{2}\right) - 2\text{SP}\left(\frac{1 - \cos \theta}{2}\right) - 4v'^2 - 4u'^2 + 4w'^2 + 4v' - 8v'w' + 8v'u' - \frac{2}{3}\pi^2 \right]. \quad (8.3)$$

u', v', w' have been defined in (6.12) and $\text{SP}(x)$ is the Spence function defined in appendix D (for completeness we give there an expression for $f(S, T, U)$ which is also valid in the low energy region). The value of q_m is usually determined from the threshold energy necessary to trigger the detectors of the outgoing particles or equivalently from the requirement that their tracks form an angle less than some maximum acollinearity angle. We are not including hard photon bremsstrahlung and this can be reasonable provided that the anisotropic part of the photon phase space can be neglected. This seems [2] a good approximation when q_m is not more

Fig. 9. Some bremsstrahlung diagrams.

than 10% of the beam energy and this corresponds [2] in our case to a maximum acollinearity angle of about 6° . We know from the analysis of ref. [4] that in this experimental situation we have to expect relevant negative corrections to the lowest-order cross section, and this is indeed the case for both pure QED and total cross section. Some remarks should be made about our assumptions.

The calculation of radiative corrections around a resonance is complicated due to the fact that even if the energy is larger than the one required to create the resonant state, the incoming electrons can lose part of their energy by bremsstrahlung and as an effect the cross section is shifted backward to the peak. This radiative tail determines an asymmetric structure of the resonance and our approximation cannot well reproduce such an effect.

Nevertheless the same soft photon approximation can be improved by realizing that if W is the total energy in the centre of mass and q is the momentum of the emitted photon then the energy available to create a given resonance of mass M_0 is $W_R \sim W - q$ so that one should evaluate the resonant part of the amplitude at this lower value when the contribution of the bremsstrahlung from the incoming particles is taken into account. This effect has been introduced in ref. [13], and it becomes quite relevant in the region where $W - M_0 \sim \Gamma$ (Γ being the width of the resonance). On the other hand, exact calculations in the resonance region cannot be performed without introducing a finite width for the unstable particle which can be deduced from the imaginary part of its self-energy. As a consequence the lowest-order amplitude gets complex and therefore we should also take into account the imaginary part of all one-loop diagrams.

Since the form factors we have been using in the calculation of radiative corrections are not suitable for the use of complex masses we have also preferred not to go beyond the standard soft photon approximation in our analysis of the bremsstrahlung.

9. Renormalization

There are three free parameters in the model, namely g , M and s_w^2 , which have to be fixed by comparison with the experimental data in order to determine unambiguously the finite part of the ultraviolet divergent diagrams. Therefore one should add radiative corrections to relations (1.1)–(1.3) in order to derive the counter terms δs_w^2 , δg , δM^2 from three known physical processes like neutral currents reactions, Coulomb scattering and μ -decay. We found it easier to derive δs_w^2 from the vector boson self-energies as it is explained below (which amounts to assume the vector boson masses as physical data) while δg could have been deduced for example from the relation

$$\sqrt{\frac{1}{2}}G_F = \frac{g^2}{8M^2}[1 + O(g^2)],$$

by calculating the quantum corrections to μ -decay, or alternatively by the corresponding expression for low-energy Coulomb scattering including weak corrections.

Since we have not carried out these calculations we are left with the renormalization of the coupling constant which in principle can be determined by choosing a suitable subtraction point for our cross section. The renormalization of the masses is as usual preliminary and the calculation of the vector boson self-energies on their mass-shell yields the relations

$$M^2 = M_u^2 - \delta M^2, \quad M_0^2 = M_{0u}^2 - \delta M_0^2, \quad (9.1)$$

(where the subscript u refers to unrenormalized quantities) from which the counter term δs_w^2 can be deduced. By putting

$$c_w^2 = c_{wu}^2 - \delta c_w^2,$$

and using the relation between the bare quantities

$$\frac{M_u^2}{M_{0u}^2} = c_{wu}^2,$$

we obtain

$$\delta s_w^2 = -c_w^2 \left[\frac{\delta M^2}{M^2} - \frac{\delta M_0^2}{M_0^2} \right]. \quad (9.2)$$

In this way the renormalization of the weak mixing angle is fixed. Further if we write

$$g = g_u[1 + \delta g g_u^2], \quad s_w = s_{wu}[1 + \delta s_w g_u^2], \quad (9.3)$$

by comparison with

$$s_w^2 = s_{wu}^2 - \delta s_w^2,$$

we get

$$\delta s_w = -\frac{1}{2} \frac{\delta s_w^2}{s_w^2 g^2}.$$

To get insight in the determination of δg we first look at pure QED, i.e., we consider the case in which only electrons, photons and muons (in the self-energy of the photon) contribute to our process. In this case we know that in the relation for the electric charge,

$$e = e_u[1 + \delta e e_u^2], \quad (9.4)$$

δe can be derived from the photon self-energy,

$$\left[\delta_{\mu\nu} - \frac{K_\mu K_\nu}{K^2} \right] \Pi(K^2),$$

by means of the relation

$$\delta e = \frac{1}{2}\Pi^{(1)}, \quad \text{with} \quad \Pi^{(1)} = \frac{\partial \Pi}{\partial K^2} \Big|_{K^2=0},$$

so that (omitting the infinite piece)

$$\delta e = \frac{1}{24\pi^2} [\ln(m^2) + \ln(m_\mu^2)]. \quad (9.5)$$

Since the perturbative expansion is carried out in powers of the physical coupling constant (the really small quantity) we can write symbolically the cross section for our process in the form

$$\frac{d\sigma^{\text{QED}}}{d\Omega} = e^4 A_0^{\text{QED}} + e^6 A_1^{\text{QED}}. \quad (9.6)$$

Then from (9.4) and (9.5) we see that we have to add the counter term

$$-4\delta e \cdot e^6 A_0^{\text{QED}} \quad (9.7)$$

to the contribution coming from lowest-order and from one-loop diagrams in order to fix the finite part of our cross section up to order e^6 .

When weak interactions are included we can write a similar relation

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^0}{d\Omega} + \frac{d\sigma^{(1)}}{d\Omega} + \frac{d\sigma^{\text{ren}}}{d\Omega},$$

and since $d\sigma^0/d\Omega$ is a function of s_w^2 only

$$\frac{d\sigma^{\text{ren}}}{d\Omega} = \left[\delta s_w^2 \frac{\partial}{\partial s_w^2} - 4\delta g \cdot g^2 \right] \frac{d\sigma^0}{d\Omega} \quad (9.8)$$

(we recall that δs_w^2 is of order g^2 but not δg). The expression for $(\partial/\partial s_w^2) d\sigma^0/d\Omega$ can be easily derived in terms of our auxiliary propagators and it turns out to be

$$\begin{aligned} \frac{\partial}{\partial s_w^2} \frac{d\sigma^0}{d\Omega} = & \left[\frac{g^2}{4\pi} \right]^2 \frac{1}{2S} \{ U^2 [\Delta'_{LL}(-S) + \Delta'_{LL}(-T)] [\Delta_{LL}(-S) + \Delta_{LL}(-T)] \\ & + U^2 [\Delta'_{RR}(-S) + \Delta'_{RR}(-T)] [\Delta_{RR}(-S) + \Delta_{RR}(-T)] \\ & + 2T^2 \Delta'_{RL}(-S) \Delta_{RL}(-S) + 2S^2 \Delta'_{RL}(-T) \Delta_{RL}(-T) \}, \end{aligned} \quad (9.9)$$

with

$$\begin{aligned} \Delta'_{LL}(x) &= D(x) + \left\{ \frac{2g_L}{c_w} + \frac{g_L^2}{c_w^2} \right\} \Delta(x), \\ \Delta'_{RL}(x) &= D(x) + \left\{ \frac{g_L + g_R}{c_w} + \frac{g_R g_L}{c_w^2} \right\} \Delta(x), \\ \Delta'_{RR}(x) &= D(x) + \left\{ \frac{2g_R}{c_w} + \frac{g_R^2}{c_w^2} \right\} \Delta(x). \end{aligned} \quad (9.10)$$

If we put for sake of brevity

$$\sigma_T = \frac{d\sigma}{d\Omega}, \quad \sigma_0 = \frac{d\sigma^0}{d\Omega}, \quad \sigma_1 = \frac{d\sigma^{(1)}}{d\Omega} + \delta s_w^2 \frac{\partial}{\partial s_w^2} \frac{d\sigma^0}{d\Omega}, \quad (9.11)$$

we get

$$\sigma_T = \sigma_0[1 - 4\delta g g^2] + \sigma_1, \quad (9.12)$$

and we can fix δg by choosing a suitable subtraction point. We have assumed that at low energies (but not too low in view of our approximation of neglecting the electron mass in the numerators of the various expressions) the result should be indistinguishable from pure QED. Choosing for the beam energy the value $E = 0.5$ GeV and for the scattering angle $\theta = 40^\circ$, we can write the purely electromagnetic contribution to our process in the form

$$\bar{\sigma}^{\text{QED}} = \bar{\sigma}_0^{\text{QED}}[1 + \bar{\delta}^{\text{QED}}], \quad (9.13)$$

(where the bar means that the quantity is calculated at the subtraction point). If we put the total cross section (including weak corrections) in the form

$$\bar{\sigma}_T = \bar{\sigma}_0[1 - 4\delta g \cdot g^2] + \bar{\sigma}_1,$$

since $\bar{\sigma}_0 \sim \bar{\sigma}_0^{\text{QED}}$ we get

$$\delta g = \frac{1}{4g^2} \left[\frac{\bar{\sigma}_1}{\bar{\sigma}_0} - \bar{\delta}^{\text{QED}} \right]. \quad (9.14)$$

Some remarks should be made to clarify how the renormalization actually works.

The various quantities δM^2 , δM_0^2 , δs_w^2 , δg have no physical meaning by themselves, they are even divergent (since they contain terms proportional to $1/(n-4)$) and contain dimensionful logarithms. Nevertheless a fixed set $\{\delta M^2, \delta M_0^2, \delta s_w^2, \delta g\}$ must give the correct predictions of the theory when combined with the contributions of all one-loop corrections, because if the renormalization is properly done all poles in $n=4$ should cancel and all logarithms should have as arguments dimensionless quantities.

In some sense this is a powerful tool to check our results. Indeed the form factors we have been using refer to dimensionally regularized integrals which can also contain ultraviolet divergences in the limit $n \rightarrow 4$. To realize this (whenever divergent) they contain a parameter which is not fixed and can be arbitrarily chosen. This parameter is essentially $1/(n-4)$ and therefore the ultraviolet divergences appear when we let the parameter grow.

The quantities δM^2 , δM_0^2 , δs_w^2 and hence δg contain divergent integrals and it is clear that each of them will blow up together with the divergent diagrams when the parameter goes from zero to infinity, but they have to grow in such a way to yield the same finite result.

Analogous effects manifest if one attempts to change the scale of the energy and of the masses. Again all the dimensionful logarithms coming from the divergent

diagrams do not scale properly, but when the counter terms are added the total percentage correction δ_T must be invariant under scaling in order that the total differential cross section,

$$\sigma_T = \sigma_0[1 + \delta_T],$$

scale as the inverse of an energy squared.

To finish the discussion of our renormalization procedure we would like to remark that our method is slightly different from the one adopted by the authors of ref. [14]. Apart from the difference in the subtraction points, we have chosen as a set of independent variables s_w^2 and g^2 while they choose s_w^2 and e^2 . This clarifies why, taking the derivative of the lowest-order cross section respect to s_w^2 , we derive also the purely electromagnetic piece while they do not.

10. Numerical results

Before describing the results of the calculations we have carried out, a few words are required in order to show how our formulas have been checked. Since the number of terms in the cross section is very high the program for its evaluation has been designed in such a way to pick up, whenever required, also the contribution of one single diagram. This has been achieved by introducing suitable coefficients (0 or 1) in order to switch off unwanted diagrams or sets of diagrams (*S*-channel, *T*-channel, pure QED ...).

Our results for pure QED are in perfect agreement with the ones of ref. [4]. The total contribution coming from the annihilation channel has been compared with the one of the authors of ref. [14] (once they put the masses of the outgoing muons equal to the mass of the incoming electrons) and we have found a constant shift (due to our different choice of the subtraction point) over the whole range of energies.

The cancellation of the infrared divergences has been checked also for single diagrams and the renormalization procedure is consistent as described in sect. 9.

The reduction formulas for the box diagrams and the traces of the γ -matrices in the cross section have been derived with the help of the algebraic program SCHOONSCHIP [15]. Since we have not introduced any decay width for the neutral vector boson some remarks are in order to estimate the reliability of the calculations around the resonance. Adding an imaginary part to the vector boson propagator amounts to taking into account both corrections to the lowest-order cross section and new contributions coming from the imaginary part of all one-loop diagrams. The typical size of these corrections (at least for the leading terms) is of the order

$$R = \bar{\Gamma}/\delta,$$

where $\delta = |-S + M_0^2|$ and $\bar{\Gamma} = \Gamma(S)M_0$.

Fig. 10. The various cross sections at $\theta = 60^\circ$ (units 10^{-2}): dotted curve, σ_0 ; full curve, σ_T ; dashed curve, σ^{OED} (see sect. 9).

Fig. 11. The various cross sections at $\theta = 90^\circ$ (units 10^{-3}): dotted curve, σ_0 ; full curve, σ_T ; dashed curve, σ^{OED} .

If one approximates $\Gamma(s)$ with $\Gamma(M_0) \sim 480$ MeV and $\delta \sim 2M_0|M_0 - E_{\text{cm}}|$ one finds

$$R \leq \frac{1}{8},$$

for $|M_0 - E_{\text{cm}}| \geq 2$ GeV.

Since we have used as a variable the beam energy and since $M_0 \sim 90$ GeV, we have not extended our calculation in the region $44 \text{ GeV} < E < 46 \text{ GeV}$. For this reason all the curves stop at $E = 44$ GeV and restart from $E = 46$ GeV, so that we could still maintain a linear scale.

We have calculated the differential cross sections and the radiative corrections for five values of the scattering angle $\theta = 20^\circ, 60^\circ, 90^\circ, 120^\circ, 160^\circ$ and in energy range up to 70 GeV per beam, they are shown in figs. 10–13. (We have not reported the data at 20° since even at 1 GeV from the peak the total cross section is too close to the QED predictions to be of some interest.) The dotted lines in figs.

Fig. 12. The various cross sections at $\theta = 120^\circ$ (units 10^{-3}): dotted curve, σ_0 ; full curve, σ_T ; dashed curve, σ^{QED} .

Fig. 13. The various cross sections at $\theta = 160^\circ$ (units 10^{-3}): dotted curve, σ_0 ; full curve, σ_T ; dashed curve, σ_{QED} .

10–13 correspond to the lowest-order cross section including weak interactions. The solid lines are the total differential cross sections while the dashed lines are the radiatively corrected predictions of pure QED.

In fig. 14 the absolute value of the total radiative corrections at 120° and 160° are shown. The main difference between lowest-order and total differential cross sections is due to soft photon effects, and it is roughly the same as in QED. Only near the peak does the effect of weak corrections become relevant as can be seen from fig. 14. This effect is entirely due to the pieces of one-loop diagrams which factorize into the W_0 propagator. Therefore the results are analogous to those one could obtain by adding radiative corrections to any resonating lowest-order amplitude. In particular the gauge structure implied by the SU(2) symmetry is not very relevant since the contribution coming from the charged vector bosons (which do

Fig. 14. The absolute value of the total radiative corrections δ_T (in %) for two values of the scattering angle. At smaller angles the radiative corrections present strong variations around the peak, cf. fig. 3 of ref. [13].

not give any resonating part) is always depressed by a few orders of magnitude as compared with the corrections of pure QED.

At small scattering angles the contribution coming from weak interactions is negligible with respect to the one-photon exchange of the T -channel up to the production of the W_0 . At large angles some effects are already present at $E \sim 35$ GeV but the absolute value of the correction to pure QED is very small. For very high energies ($E \sim 60$ GeV) the various cross sections decrease even faster than the QED predictions. If the value of the weak mixing angle is increased in such a way as to reduce the mass of the neutral vector boson, all the phenomenology of the weak corrections can be shifted to lower energies but even

in this case all the interesting features of the model should be concentrated in the region where the W_0 can be created as a physical particle.

Beyond this region only the possibility to have new competitive processes like $e^+e^- \rightarrow W^+W^-$ can yield an overall enhancement of the role of weak and electromagnetic interactions in the electron-positron physics.

11. Conclusions

With the values of s_w^2 and M_0 we have been using for the Weinberg model, the contribution of weak interactions to Bhabha scattering is really too small to be of experimental interest up to energies of about 35 GeV per beam. From 35 GeV to 40 GeV only scattering at large angles can give some information. For energies very close to the threshold of production of the neutral vector boson our calculations should be considered as part of an approximation to a full calculation where finite width effects and hard photon bremsstrahlung are taken into account as well.

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Appendix A

We give a list of the relevant Feynman rules of the Weinberg model. The expression for the Lagrangian in terms of the diagonal fields can be found in ref. [14].

Propagators

neutral vector boson

$$\frac{\delta_{\mu\nu}}{k^2 + M_0^2 - i\epsilon}$$

photon

$$\frac{\delta_{\mu\nu}}{k^2 - i\epsilon}$$

charged vector boson

$$\frac{\delta_{\mu\nu}}{k^2 + M^2 - i\epsilon}$$

massive leptons

$$\frac{-ik + m_e}{k^2 + m_e^2 - i\epsilon}$$

neutrinos

$$\frac{-ik}{k^2 - i\epsilon}$$

Higgs particle

$$\frac{1}{k^2 + M_z^2 - i\epsilon}$$

neutral Higgs-Kibble ghost

$$\frac{1}{k^2 M_0^2 - i\epsilon}$$

charged Higgs-Kibble ghost

$$\text{---}\phi^\pm\text{---} \quad \frac{1}{k^2 + M^2 - i\varepsilon}$$

Faddeev-Popov ghosts

$$\text{---}\eta_\pm\text{---} \quad \frac{1}{k^2 + M^2 - i\varepsilon} \quad \text{---}\eta_0\text{---} \quad \frac{1}{k^2 + M_0^2 - i\varepsilon} \quad \text{---}\psi\text{---} \quad \frac{1}{k^2 - i\varepsilon}$$

A factor $1/(2\pi)^4 i$ has to be introduced for every propagator.

Vertices

$$\begin{array}{c} \nu \\ \swarrow \\ \nu \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\mu^0 \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad -igb\gamma_\mu(1+\gamma_5)$$

$$\begin{array}{c} \nu \\ \swarrow \\ \ell \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} A_\mu \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad -igs_w\gamma_\mu$$

$$\begin{array}{c} Z \\ \swarrow \\ \phi^0 \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} p \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} q \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\mu^0 \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad \frac{1}{2}i\frac{g}{c_w}(q-p)_\mu$$

$$\begin{array}{c} Z \\ \swarrow \\ \phi^\pm \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} p \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} q \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\mu^\mp \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad \frac{1}{2}ig(q-p)_\mu$$

$$\begin{array}{c} Z \\ \swarrow \\ W_\mu^+ \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\nu^- \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad -Mg\delta_{\mu\nu}$$

$$\begin{array}{c} \phi^+ \\ \swarrow \\ \phi^- \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} p \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} q \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\mu^0 \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad -\frac{1}{2}g\frac{(s_w^2 - c_w^2)}{c_w}(p-q)_\mu$$

$$\begin{array}{c} \phi^\mp \\ \swarrow \\ W_\mu^\pm \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\nu^0 \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad \mp iMg\frac{s_w^2}{c_w}\delta_{\mu\nu}$$

$$\begin{array}{c} \nu \\ \swarrow \\ \ell \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\mu^\pm \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad \frac{ig}{2\sqrt{2}}\gamma_\mu(1+\gamma_5)$$

$$\begin{array}{c} \ell \\ \swarrow \\ \ell \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\mu^0 \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad ig\gamma_\mu(a+b\gamma_5)$$

$$\begin{array}{c} \phi^0 \\ \swarrow \\ \phi^\pm \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} p \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} q \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\mu^\mp \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad \mp \frac{1}{2}g(q-p)_\mu$$

$$\begin{array}{c} Z \\ \swarrow \\ W_\nu^0 \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} W_\mu^0 \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad -\frac{1}{2}\frac{Mg}{c_w^2}\delta_{\mu\nu} \cdot 2!$$

$$\begin{array}{c} \phi^+ \\ \swarrow \\ \phi^- \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} p \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} q \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} A_\mu \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad gs_w(p-q)_\mu$$

$$\begin{array}{c} \phi^\mp \\ \swarrow \\ W_\mu^\pm \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} A_\nu \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad \pm iMgs_w\delta_{\mu\nu}$$

$$\begin{array}{c} A_\mu \\ \swarrow \\ A_\nu \end{array} \begin{array}{c} \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} \phi^+ \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \begin{array}{c} \phi^- \\ \nearrow \\ \nearrow \\ \nearrow \end{array} \quad -g^2s_w^2\delta_{\mu\nu} \cdot 2!$$

$$-\frac{1}{8} \frac{g^2}{c_w^2} \delta_{\mu\nu} \cdot 2!2!$$

$$-\frac{1}{8} \frac{g^2}{c_w^2} \delta_{\mu\nu} \cdot 2!2!$$

$$g^2 \frac{s_w}{c_w} (s_w^2 - c_w^2) \delta_{\mu\nu}$$

$$-\frac{1}{4} g^2 \frac{(s_w^2 - c_w^2)^2}{c_w^2} \delta_{\mu\nu} \cdot 2!$$

$$-\frac{1}{4} g^2 \delta_{\mu\nu} \cdot 2!$$

$$-\frac{1}{4} g^2 \delta_{\mu\nu} \cdot 2!$$

$$-\frac{1}{2} g^2 \delta_{\mu\nu}$$

$$-g(s_w, c_w) [\delta_{\alpha\gamma} (k-q)_\beta + \delta_{\beta\alpha} (p-k)_\gamma + \delta_{\gamma\beta} (q-p)_\alpha]$$

$$-g^2 (s_w^2, c_w^2) [\delta_{\alpha\beta} \delta_{\gamma\delta} - \delta_{\alpha\gamma} \delta_{\beta\delta}] 2!$$

A factor $(2\pi)^4 i$ has to be introduced for every vertex (we have indicated explicitly the combinatorial factors) and the usual minus sign has to be associated with the FP ghosts closed loops.

Appendix B

The form factors

(a) 1-point function

$$A(M) \equiv \int d^n q \frac{1}{q^2 + M^2}. \quad (\text{B.1})$$

(b) 2-point functions

$$B_0(k^2, M_1, M_2) \equiv \int d^n q \frac{1}{(q^2 + M_1^2)((q+k)^2 + M_2^2)}, \quad (\text{B.2})$$

$$B_\mu(k, M_1, M_2) \equiv \int d^n q \frac{q_\mu}{(q^2 + M_1^2)((q+k)^2 + M_2^2)} \equiv B_1 \cdot k_\mu, \quad (\text{B.3})$$

$$B_{\mu\nu}(k, M_1, M_2) \equiv \int d^n q \frac{q_\mu q_\nu}{(q^2 + M_1^2)((q+k)^2 + M_2^2)} \equiv B_{21} k_\mu k_\nu + B_{22} \delta_{\mu\nu}. \quad (\text{B.4})$$

(c) 3-point functions

$$C_0(P^2, k^2, (P+k)^2, M_1, M_2, M_3) \equiv \int d^4q \frac{1}{(1)(2)(3)},$$

$$(1) = q^2 + M_1^2, \quad (2) = (q+P)^2 + M_2^2, \quad (3) = (q+P+k)^2 + M_3^2, \quad (\text{B.5})$$

$$C_\mu(P, k, M_1, M_2, M_3) \equiv \int d^4q \frac{q_\mu}{(1)(2)(3)} \equiv C_{11}P_\mu + C_{12}k_\mu, \quad (\text{B.6})$$

$$C_{\mu\nu}(P, k, M_1, M_2, M_3) \equiv \int d^4q \frac{q_\mu q_\nu}{(1)(2)(3)}$$

$$\equiv C_{21}P_\mu P_\nu + C_{22}k_\mu k_\nu + C_{23}(P_\mu k_\nu + P_\nu k_\mu) + C_{24}\delta_{\mu\nu}. \quad (\text{B.7})$$

(d) 4-point functions

$$D_0(P_1^2, P_2^2, P_3^2, P_4^2, (P_1+P_2)^2, (P_2+P_3)^2, M_1, M_2, M_3, M_4)$$

$$\equiv \int d^4q \frac{1}{(1)(2)(3)(4)}, \quad (\text{B.8})$$

$$(1) = q^2 + M_1^2, \quad (2) = (q+p_1)^2 + M_2^2, \quad (3) = (q+P_1+P_2)^2 + M_3^2,$$

$$(4) = (q+P_1+P_2+P_3)^2 + M_4^2,$$

(see fig. 8).

We define also

$$D_\mu(P_1, P_2, P_3, M_1, M_2, M_3, M_4) \equiv \int d^4q \frac{q_\mu}{(1)(2)(3)(4)}$$

$$\equiv D_{11}P_1^\mu + D_{12}P_2^\mu + D_{13}P_3^\mu, \quad (\text{B.9})$$

$$D_{\mu\nu}(P_1, P_2, P_3, M_1, M_2, M_3, M_4) \equiv \int d^4q \frac{q_\mu q_\nu}{(1)(2)(3)(4)}$$

$$\equiv D_{21}P_1^\mu P_1^\nu + D_{22}P_2^\mu P_2^\nu + D_{23}P_3^\mu P_3^\nu$$

$$+ D_{24}(P_1^\mu P_2^\nu + P_2^\mu P_1^\nu)$$

$$+ D_{25}(P_1^\mu P_3^\nu + P_3^\mu P_1^\nu)$$

$$+ D_{26}(P_2^\mu P_3^\nu + P_3^\mu P_2^\nu) + \delta_{\mu\nu} D_{27}. \quad (\text{B.10})$$

Also

$$F_1 \equiv -2[D_{12} - D_{13} + D_{24} - D_{25}],$$

$$F_2 \equiv D_0 + D_{11} - D_{12} + D_{13} + 2D_{25} - 2D_{26},$$

$$F_3 \equiv F_2 - D_{25} + D_{26},$$

$$F_4 \equiv D_{27}. \quad (\text{B.11})$$

The expressions of the various form factors in terms of the scalar functions A , B_0 , C_0 , D_0 can be found in ref. [14], the expressions for A , B_0 , C_0 and D_0 in ref. [8] (our form factors are not divided by $i\pi^2$ as in ref. [14]).

Appendix C

Self-energy diagrams

For simplicity we define the following notations:

$$L_{\mu\nu}(k, M_1, M_2) \equiv 2k_\mu k_\nu (B_{21} + B_1) + 2\delta_{\mu\nu} B_{22} - k^2 \delta_{\mu\nu} B_1 - A(M_2) \delta_{\mu\nu},$$

$$\phi_{\mu\nu}(k, M_1, M_2) \equiv k_\mu k_\nu (B_0 + 4B_{21} + 4B_1) + 4\delta_{\mu\nu} B_{22},$$

$$V_{\mu\nu}(k, M_1, M_2) \equiv k_\mu k_\nu (10B_1 + 10B_{21} - 2B_0) + \delta_{\mu\nu} (2A(M_2) - 2M_1^2 B_0 + 10B_{22} \\ + 5k^2 B_0 + 2k^2 B_1) - (k_\mu k_\nu - k^2 \delta_{\mu\nu}) \frac{2}{3} i\pi^2 + 2i\pi^2 (M_1^2 + M_2^2) \delta_{\mu\nu},$$

$$\Gamma_{\mu\nu}(k, M_1, M_2) \equiv k_\mu k_\nu (B_{21} + B_1) + \delta_{\mu\nu} B_{22},$$

$$T_{\mu\nu}(M) = (4i\pi^2 M^2 + 6A(M)) \delta_{\mu\nu},$$

and all the B -functions depend on k^2 , M_1 , M_2 .

The self-energy diagrams are the following (all the diagrams contain their combinatorial factors as explained in ref. [10]).

(i) *Photon*

$$M_1 = m, m_\mu, \quad M_2 = m, m_\mu, \\ -4g^2 s_w^2 L_{\mu\nu}(k, M_1, M_2),$$

$$g^2 s_w^2 \phi_{\mu\nu}(k, M, M),$$

$$-2g^2 s_w^2 \delta_{\mu\nu} A(M),$$

$$g^2 s_w^2 V_{\mu\nu}(k, M, M),$$

$$-g^2 s_w^2 T_{\mu\nu}(M),$$

$$2g^2 s_w^2 M^2 B_0(k^2, M, M) \delta_{\mu\nu}, \quad (\text{sum of the two diagrams}),$$

$$-2g^2 s_w^2 \Gamma_{\mu\nu}(k, M, M), \quad (\text{sum of the two diagrams}),$$

$$\Pi_{\mu\nu}^{22} = g^2 \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] F_{22}(k^2), \quad F_{22}(0) = 0. \quad (\text{C.1})$$

(ii) $A_\mu - W_\nu^0$ transition

$$M_1 = m, m_\mu, \quad M_2 = m, m_\mu, \\ 2g^2 s_w (g_L + g_R) L_{\mu\nu}(k, M_1, M_2),$$

$$g^2 s_w \frac{(c_w^2 - s_w^2)}{2c_w} \phi_{\mu\nu}(k, M, M),$$

$$-g^2 s_w \frac{(c_w^2 - s_w^2)}{c_w} A(M) \delta_{\mu\nu},$$

$$g^2 s_w c_w V_{\mu\nu}(k, M, M),$$

$$-g^2 s_w c_w T_{\mu\nu}(M),$$

$$\begin{aligned}
& \text{Diagram 1: } \mu \text{ (solid line, } k) \text{ and } \bar{\nu} \text{ (dashed line, } k) \text{ connected by a } W^\pm \text{ loop.} & 2g^2 s_w c_w [M^2 - M_0^2] B_0(k^2, M, M), & \text{(sum of the two diagrams)} \\
& \text{Diagram 2: } \mu \text{ (solid line, } k) \text{ and } \bar{\nu} \text{ (dashed line, } k) \text{ connected by a } \eta_\pm \text{ loop.} & -2g^2 s_w c_w \Gamma_{\mu\nu}(k, M, M), & \text{(sum of the two diagrams),}
\end{aligned}$$

$$\Pi_{\mu\nu}^{12} = g^2 [k_\mu k_\nu A_{12}(k^2) + \delta_{\mu\nu} F_{12}(k^2)]. \quad (\text{C.2})$$

The only piece in F_{12} which is not proportional to k^2 is $-2g^2 s_w c_w M_0^2 B_0$ coming from the diagrams with $\phi^\pm$ and $W^\mp$. This contribution is crucial to show the cancellation of the terms γ_5 in the photon-electron coupling at low energy. For a derivation see ref. [14].

(iii) Neutral vector boson self-energy

$$\begin{aligned}
& \text{Diagram 1: } \mu \text{ (dashed line, } k) \text{ and } \bar{\nu} \text{ (dashed line, } k) \text{ connected by an } e^+e^- \text{ loop.} & M_1 = m, m_\mu, \quad M_2 = m, m_\mu, \\
& & -4g^2(a^2 + b^2)L_{\mu\nu}(k, M_1, M_2), \\
& \text{Diagram 2: } \mu \text{ (dashed line, } k) \text{ and } \bar{\nu} \text{ (dashed line, } k) \text{ connected by a } \bar{\nu}_e \nu_e \text{ loop.} & -8g^2 b^2 L_{\mu\nu}(k, 0, 0), & \text{(sum of the two diagrams),} \\
& \text{Diagram 3: } \mu \text{ (dashed line, } k) \text{ and } \bar{\nu} \text{ (dashed line, } k) \text{ connected by a } \phi^+ \phi^- \text{ loop.} & g^2 \frac{(s_w^2 - c_w^2)^2}{4c_w^2} \phi_{\mu\nu}(k, M, M), \\
& \text{Diagram 4: } \mu \text{ (dashed line, } k) \text{ and } \bar{\nu} \text{ (dashed line, } k) \text{ connected by a } \phi^+ \phi^- \text{ loop.} & -g^2 \frac{(s_w^2 - c_w^2)^2}{2c_w^2} \delta_{\mu\nu} A(M), \\
& \text{Diagram 5: } \mu \text{ (dashed line, } k) \text{ and } \bar{\nu} \text{ (dashed line, } k) \text{ connected by a } Z \text{ loop.} & \frac{g^2}{4c_w^2} \phi_{\mu\nu}(k, M_Z, M_0),
\end{aligned}$$

$$\frac{g^2 M_0^2}{c_w^2} \delta_{\mu\nu} B_0(k^2, M_Z, M_0),$$

$$-\frac{g^2}{4c_w^2} A(M_Z) \delta_{\mu\nu},$$

$$-\frac{g^2}{4c_w^2} A(M_0) \delta_{\mu\nu},$$

$$g^2 c_w^2 V_{\mu\nu}(k, M, M),$$

$$-g^2 c_w^2 T_{\mu\nu}(M),$$

$$2g^2 M^2 \frac{s_w^4}{c_w^2} \delta_{\mu\nu} B_0(k^2, M, M), \quad (\text{sum of the two diagrams}),$$

$$-2g^2 c_w^2 \Gamma_{\mu\nu}(k, M, M), \quad (\text{sum of the two diagrams}),$$

$$\Pi_{\mu\nu}^{11} = g^2 [k_\mu k_\nu A_{11}(k^2) + \delta_{\mu\nu} F_{11}(k^2)],$$

$$\tilde{F}_{11} = F_{11}/(2\pi)^4 i, \quad \delta M_0^2 = g^2 \text{Real} [\tilde{F}_{11}(-M_0^2)]. \quad (\text{C.3})$$

(iv) Charged vector boson self-energy

$$M_1 = 0, \quad M_2 = m, m_\mu, \\ -g^2 L_{\mu\nu}(k, M_1, M_2),$$

$$\frac{1}{4}g^2 \phi_{\mu\nu}(k, M_0, M),$$

$$\frac{1}{4}g^2 \phi_{\mu\nu}(k, M_Z, M),$$

$$g^2 M^2 \delta_{\mu\nu} B_0(k^2, M_Z, M),$$

$$g^2 s_w^2 M^2 \delta_{\mu\nu} B_0(k^2, 0, M),$$

$$g^2 M^2 \frac{s_w^2}{c_w^2} \delta_{\mu\nu} B_0(k^2, M_0, M),$$

$$-\frac{1}{4}g^2 \delta_{\mu\nu} A(M_Z),$$

$$-\frac{1}{4}g^2 \delta_{\mu\nu} A(M_0),$$

$$-\frac{1}{2}g^2 \delta_{\mu\nu} A(M),$$

$$\begin{array}{c} \text{A} \\ \text{---} \frac{k}{\mu} \text{---} \text{---} \frac{k}{\nu} \text{---} \\ \text{W} \end{array} \quad g^2 s_w^2 V_{\mu\nu}(k, 0, M),$$

$$\begin{array}{c} \text{W}_0 \\ \text{---} \mu \text{---} \text{---} \nu \text{---} \\ \text{W} \end{array} \quad g^2 c_w^2 V_{\mu\nu}(k, M_0, M),$$

$$\begin{array}{c} \text{W}^+ \quad \text{W}^- \\ \text{---} \frac{k}{\mu} \text{---} \text{---} \frac{k}{\nu} \text{---} \end{array} \quad -\frac{1}{2} g^2 T_{\mu\nu}(M),$$

$$\begin{array}{c} \text{W}_0 \\ \text{---} \frac{k}{\mu} \text{---} \text{---} \frac{k}{\nu} \text{---} \end{array} \quad -\frac{1}{2} g^2 T_{\mu\nu}(M_0),$$

$$\begin{array}{c} \eta_0 \\ \text{---} \frac{k}{\mu} \text{---} \text{---} \frac{k}{\nu} \text{---} \\ \eta_+ \end{array} \quad -g^2 c_w^2 \Gamma_{\mu\nu}(k, M_0, M),$$

$$\begin{array}{c} \eta_- \\ \text{---} \frac{k}{\mu} \text{---} \text{---} \frac{k}{\nu} \text{---} \\ \eta_0 \end{array} \quad -g^2 c_w^2 \Gamma_{\mu\nu}(k, M, M_0),$$

$$\begin{array}{c} \psi \\ \text{---} \frac{k}{\mu} \text{---} \text{---} \frac{k}{\nu} \text{---} \\ \eta_+ \end{array} \quad -g^2 s_w^2 \Gamma_{\mu\nu}(k, 0, M),$$

$$\begin{array}{c} \eta_- \\ \text{---} \frac{k}{\mu} \text{---} \text{---} \frac{k}{\nu} \text{---} \\ \psi \end{array} \quad -g^2 s_w^2 \Gamma_{\mu\nu}(k, M, 0),$$

$$\Pi_{\mu\nu}^\pm = g^2 (k_\mu k_\nu A_+(k^2) + \delta_{\mu\nu} F_+(k^2))$$

$$\tilde{F}_+ = F_+ / (2\pi)^4 i, \quad \delta M^2 = g^2 \text{Real} [\tilde{F}_+(-M^2)].$$

(C.4)

(v) *Self-energy of the electron*

$$\tilde{\Sigma} = ik[A + B\gamma_5] + m_u C, \quad \tilde{\Sigma} = \Sigma/(2\pi)^4 i.$$

(see sect. 5)

$$M_1 = \lambda, \quad M_2 = m,$$

$$\text{contribution to } A: (ig)^2 s_w^2 [-i\pi^2 + 2B_1 + 2B_0]/(2\pi)^4 i,$$

$$\text{contribution to } B: 0,$$

$$\text{contribution to } C: (ig)^2 s_w^2 [4B_0]/(2\pi)^4 i.$$

$$M_1 = M_0, \quad M_2 = m,$$

$$\text{contribution to } A: (ig)^2 (a^2 + b^2) [-i\pi^2 + 2B_1 + 2B_0]/(2\pi)^4 i,$$

$$\text{contribution to } B: (ig)^2 2ab [-i\pi^2 + 2B_1 + 2B_0]/(2\pi)^4 i,$$

$$\text{contribution to } C: (ig)^2 (a^2 - b^2) [4B_0]/(2\pi)^4 i.$$

$$M_1 = M, \quad M_2 = 0,$$

$$\text{contribution to } A: (ig)^2 \frac{1}{4} [-i\pi^2 + 2B_1 + 2B_0]/(2\pi)^4 i,$$

$$\text{contribution to } B: (ig)^2 \frac{1}{4} [-i\pi^2 + 2B_1 + 2B_0]/(2\pi)^4 i,$$

$$\text{contribution to } C: 0.$$

λ is introduced to regularize the usual infrared divergence in the derivative of the electron self-energy on-mass-shell. Indeed, for example,

$$B'_0 = \frac{\partial}{\partial k^2} B_0(k^2, \lambda, m) \Big|_{k^2 = -m^2} = -i\pi^2 \int_0^1 dx \frac{x(1-x)}{m^2 x^2 + \lambda^2 (1-x)},$$

from which we get

$$B'_0 = \frac{i\pi^2}{2m^2} \left[2 - \ln \frac{m^2}{\lambda^2} \right].$$

Appendix D

The bremsstrahlung integral

We give in this section an expression for Bhabha scattering of the function $f(S, T, U)$ defined in (8.2). Defining

$$S = 4E^2, \quad T = 2(E^2 - m^2)(\cos \theta - 1), \quad U = -S - T + 4m^2,$$

$$R(x) = |x| \sqrt{1 - \frac{4m^2}{x}}, \quad \rho(x) = \frac{1}{2m^2} [-x + 2m^2 + R(x)], \quad (\text{D.1})$$

we can put $f(S, T, U)$ in the following form:

$$\begin{aligned} f(S, T, U) = & 2 \left\{ (2m^2 - T) \left[\pi \ln \frac{4q_m^2}{\lambda^2} \theta(T) + \alpha(T, S) \right] \right. \\ & + (S - 2m^2) \left[-\pi \ln \frac{4q_m^2}{\lambda^2} \theta(S) + \beta(S) \right] \\ & \left. + (U - 2m^2) \left[\pi \ln \frac{4q_m^2}{\lambda^2} \theta(U) + \alpha(U, S) \right] - 2m^2 \left[\frac{\pi}{m^2} \ln \frac{4q_m^2}{\lambda^2} + \frac{\pi}{m^2} \gamma(S) \right] \right\}, \end{aligned} \quad (D.2)$$

where

$$\theta(x) = \frac{1}{R(x)} \ln \rho^2(x), \quad (D.3)$$

$$\gamma(S) = \frac{S}{R(S)} \ln \frac{S - R(S)}{S + R(S)}, \quad (D.4)$$

$$\beta(S) = -\frac{\pi}{R(S)} \left[\ln^2 \left(\frac{S - R(S)}{S + R(S)} \right) + 4 \text{SP} \left(\frac{2R(S)}{S + R(S)} \right) \right], \quad (D.5)$$

$$\begin{aligned} \alpha(x, S) = & \frac{2\pi}{R(x)} \left\{ \text{SP} \left(1 - \frac{S + R(S)}{2m^2} \frac{\rho(x)}{\rho(x) + 1} \right) + \text{SP} \left(1 - \frac{S - R(S)}{2m^2} \frac{\rho(x)}{\rho(x) + 1} \right) \right. \\ & \left. - \text{SP} \left(1 - \frac{S + R(S)}{2m^2} \frac{1}{\rho(x) + 1} \right) - \text{SP} \left(1 - \frac{S - R(S)}{2m^2} \frac{1}{\rho(x) + 1} \right) \right\}, \end{aligned} \quad (D.6)$$

and $\text{SP}(x)$ is the Spence function defined by

$$\text{SP}(x) = - \int_0^x dt \frac{1}{t} \ln(1-t). \quad (D.7)$$

The expression for $\beta(S)$ can be found in ref. [2] while our formula for $\alpha(x, S)$ has been adapted from the results of ref. [8].

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