LETTER TO THE EDITOR

The angular correlation of two gamma quanta emitted by aligned radioactive nuclei

In a preceding note we calculated the angular distribution and polarization of γ -quanta emitted by aligned nuclei¹⁾. Other phenomena can be studied, if two γ -quanta are emitted in succession. We have calculated the way in which the law of angular correlation for nuclei with random orientation is changed, if the nuclei are aligned.

If we have two dipole transitions in succession with $j_i \rightarrow j_i - 1 \rightarrow j_i - 2$ the law of angular correlation becomes

$$\begin{aligned}
 W(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) = & 9 \{ [13/30 - (4/7)N_2f_2 + \frac{1}{8}N_4f_4] + \\
 & + [1/30 - \frac{1}{7}N_2f_2 + \frac{1}{4}N_4f_4] (\mathbf{k}_1 \cdot \mathbf{k}_2)^2 + \\
 & + [(6/7)N_2f_2 - (5/8)N_4f_4] [(\mathbf{k}_1 \cdot \boldsymbol{\eta})^2 + (\mathbf{k}_2 \cdot \boldsymbol{\eta})^2] + \\
 & + (35/8)N_4f_4 (\mathbf{k}_1 \cdot \boldsymbol{\eta})^2 (\mathbf{k}_2 \cdot \boldsymbol{\eta})^2 + \\
 & + [(3/7)N_2f_2 - (5/2)N_4f_4] (\mathbf{k}_1 \cdot \boldsymbol{\eta}) (\mathbf{k}_2 \cdot \boldsymbol{\eta}) (\mathbf{k}_1 \cdot \mathbf{k}_2) \}. \quad (1)
 \end{aligned}$$

$\mathbf{k}_1$ and $\mathbf{k}_2$ are the directions of emission of the first and second γ -quanta; $\boldsymbol{\eta}$ is the axis of orientation; other notations are the same as in¹⁾. For nuclei with random orientation $f_2 = f_4 = 0$, and (1) becomes the well-known angular correlation law. For total alignment (1) becomes

$$W(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) = (9/4) [1 + (\mathbf{k}_1 \cdot \boldsymbol{\eta})^2] [1 + (\mathbf{k}_2 \cdot \boldsymbol{\eta})^2] = W(\mathbf{k}_1, \boldsymbol{\eta}) W(\mathbf{k}_2, \boldsymbol{\eta}) \quad (2)$$

$W(\mathbf{k}_1, \boldsymbol{\eta})$ and $W(\mathbf{k}_2, \boldsymbol{\eta})$ are the angular distributions of the radiations separately ($W(\vartheta) = (3/2) (1 + \cos^2 \vartheta)$ for total alignment). Hence for total alignment there is no angular correlation.

For the case of ^{60}Co , where alignment has been studied experimentally, we have two quadrupole transitions in succession with $j_i \rightarrow j_i - 2 \rightarrow j_i - 4$ (after the beta transition; $j_i = 4$). For this case of ^{60}Co we have calculated for the law of angular correlation

$$\begin{aligned}
 W(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) = & 25 \{ [(16/105) + (32/77)N_2f_2 - (201/286)N_4f_4 - (1/10)N_6f_6 + (3/32)N_8f_8] + \\
 & + [(2/105) - (8/77)N_2f_2 + (54/143)N_4f_4 - (4/5)N_6f_6 + (3/4)N_8f_8] [(\mathbf{k}_1 \cdot \mathbf{k}_2)^2 + \frac{1}{3}(\mathbf{k}_1 \cdot \mathbf{k}_2)^4] + \\
 & + [-(48/77)N_2f_2 + (1005/286)N_4f_4 + (21/20)N_6f_6 - (27/16)N_8f_8] [(\mathbf{k}_1 \cdot \boldsymbol{\eta})^2 + (\mathbf{k}_2 \cdot \boldsymbol{\eta})^2] + \\
 & + [(6/77)N_2f_2 - (135/143)N_4f_4 + (21/5)N_6f_6 - (27/4)N_8f_8] F_1(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) + \\
 & + [-(1225/286)N_4f_4 - (21/20)N_6f_6 + (99/32)N_8f_8] [(\mathbf{k}_1 \cdot \boldsymbol{\eta})^4 + (\mathbf{k}_2 \cdot \boldsymbol{\eta})^4] + \\
 & + [(35/143)N_4f_4 - (14/5)N_6f_6 + (33/4)N_8f_8] F_2(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) + \\
 & + [(77/5)N_6f_6 - (429/4)N_8f_8] F_3(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) + (6435/32)N_8f_8 (\mathbf{k}_1 \cdot \boldsymbol{\eta})^4 (\mathbf{k}_2 \cdot \boldsymbol{\eta})^4 \}. \quad (3)
 \end{aligned}$$

with

$$\begin{aligned}
 F_1(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) &= (\mathbf{k}_1 \cdot \mathbf{k}_2) [2(\mathbf{k}_1 \cdot \boldsymbol{\eta})(\mathbf{k}_2 \cdot \boldsymbol{\eta}) + \frac{4}{3}(\mathbf{k}_1 \cdot \boldsymbol{\eta})(\mathbf{k}_2 \cdot \boldsymbol{\eta})(\mathbf{k}_1 \cdot \mathbf{k}_2)^2 + \\
 &\quad + (\mathbf{k}_1 \cdot \boldsymbol{\eta})^2(\mathbf{k}_1 \cdot \mathbf{k}_2) + (\mathbf{k}_2 \cdot \boldsymbol{\eta})^2(\mathbf{k}_1 \cdot \mathbf{k}_2)], \\
 F_2(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) &= (\mathbf{k}_1 \cdot \boldsymbol{\eta})(\mathbf{k}_2 \cdot \boldsymbol{\eta})[9(\mathbf{k}_1 \cdot \boldsymbol{\eta})(\mathbf{k}_2 \cdot \boldsymbol{\eta})(\mathbf{k}_1 \cdot \mathbf{k}_2)^2 + (9/2)(\mathbf{k}_1 \cdot \boldsymbol{\eta})(\mathbf{k}_2 \cdot \boldsymbol{\eta}) + \\
 &\quad + 6(\mathbf{k}_1 \cdot \boldsymbol{\eta})^2(\mathbf{k}_1 \cdot \mathbf{k}_2) + 6(\mathbf{k}_2 \cdot \boldsymbol{\eta})^2(\mathbf{k}_1 \cdot \mathbf{k}_2)], \\
 F_3(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) &= (\mathbf{k}_1 \cdot \boldsymbol{\eta})^2(\mathbf{k}_2 \cdot \boldsymbol{\eta})^2[2(\mathbf{k}_1 \cdot \boldsymbol{\eta})(\mathbf{k}_2 \cdot \boldsymbol{\eta})(\mathbf{k}_1 \cdot \mathbf{k}_2) + \frac{3}{4}(\mathbf{k}_1 \cdot \boldsymbol{\eta})^2 + \frac{3}{4}(\mathbf{k}_2 \cdot \boldsymbol{\eta})^2], \quad (4) \\
 N_6 &= j^5/(j-2)(j-1)(2j-1)(2j-3)(2j-5), \\
 N_8 &= j^7/(j-3)(j-2)(j-1)(2j-1)(2j-3)(2j-5)(2j-7), \quad (5) \\
 f_6 &= (1/j^6) [\Sigma_m m^6 a_m - (5/11)(3j^2 + 3j - 7) \Sigma_m m^4 a_m + \\
 &\quad + (1/11)(5j^4 + 10j^3 - 20j^2 - 25j + 14) \Sigma_m m^2 a_m - \\
 &\quad - (5/231)j(j+1)(j-1)(j+2)(j-2)(j+3)], \\
 f_8 &= (1/j^8) [\Sigma_m m^8 a_m - (14/15)(2j^2 + 2j - 9) \Sigma_m m^6 a_m + (7/39)(6j^4 + 12j^3 - \\
 &\quad - 50j^2 - 56j + 81) \Sigma_m m^4 a_m - (7/2145)(60j^6 + 180j^5 - 690j^4 - 1680j^3 + \\
 &\quad + 1958j^2 + 2828j - 9132/7) \Sigma_m m^2 a_m + \\
 &\quad + (7/1287)j(j+1)(j-1)(j+2)(j-2)(j+3)(j-3)(j+4)]. \quad (6)
 \end{aligned}$$

Just as in the dipole-dipole case we get the ordinary angular correlation formula for nuclei with random orientation, for $f_2 = f_4 = f_6 = f_8 = 0$. For total alignment (3) becomes

$$W(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) = (25/4) [1 - (\mathbf{k}_1 \cdot \boldsymbol{\eta})^4] [1 - (\mathbf{k}_2 \cdot \boldsymbol{\eta})^4] = W(\mathbf{k}_1, \boldsymbol{\eta}) W(\mathbf{k}_2, \boldsymbol{\eta}). \quad (7)$$

Hence in this case there is no angular correlation left ($W(\theta) = (5/2)(1 - \cos^4 \theta)$ for total alignment), as in the case of dipole-dipole transitions. In general it can be shown that no angular correlation will be left for total alignment, if we have a 2^{L_1} -pole and 2^{L_2} -pole radiation in succession and if

$$j_i \rightarrow j_i - L_1 \rightarrow j_i - L_1 - L_2.$$

The normalization of (1) and (3) is such that

$$\int W(\mathbf{k}_1, \mathbf{k}_2, \boldsymbol{\eta}) d\Omega_1 d\Omega_2 = (8\pi)^2 \quad (8)$$

A full account of the theory and suggestions for experiments will appear later in *Physica*. We wish to thank Prof. S. R. de Groot for his interest and for valuable discussions.

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- 1) Tolhoek, H. A. and Cox, J. A. M., Angular distribution and polarization of gamma radiation emitted by aligned radioactive nuclei, *Physica* **18** (1952) 357—358.