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The Use of an Analog Computer in a Circulation Model

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THE USE OF computers in the study of the normal and abnormal circulatory system in humans has been introduced in several ways and for very different reasons.

One reason was to create the possibility of handling vast quantities of data in a reasonably short time, as is done at present in evaluating electro- and vectorcardiograms,[1] or to handle multidimensional information to arrive quickly at the most probable diagnosis by taking into account many aspects concerning each patient, such as the electrocardiogram, phonocardiogram, blood pressures, temperature, age, etc.[2,3] For such purposes only digital computers are used at the present time.

Another reason for using computers is for solving complicated equations or for evaluating systems that are described by many equations with many unknowns, where solution by hand would require too much time. Such problems can, in many cases, be solved successfully by programming the problem on a general purpose or special purpose analog computer, which then acts as a model of the system under investigation. In some cases digital computation is preferred, while sometimes mixed computer usage may be advantageous.

In the study of the circulatory system, the digital, the analog, and the mixed approach are used. Examples of such studies are the investigation of the pulse wave velocity in infinitely long tubes,[4] the study of regulatory mechanisms,[5,6] the determintion of arterial wall properties.[7]

In this paper, the development of a special purpose analog computer for the study of human hemodynamics will be described and the reason for its development will be set forth.

NEED FOR A COMPUTER

Several years ago the attempt was made to find a quantitative interpretation for the ultra-low frequency displacement, velocity, and acceleration (force) ballistocardiograms recorded in the head-foot direction. The quantities recorded here represent the displacement, velocity, and acceleration of the internal center of gravity of the subject, i.e., with respect to the subject's own

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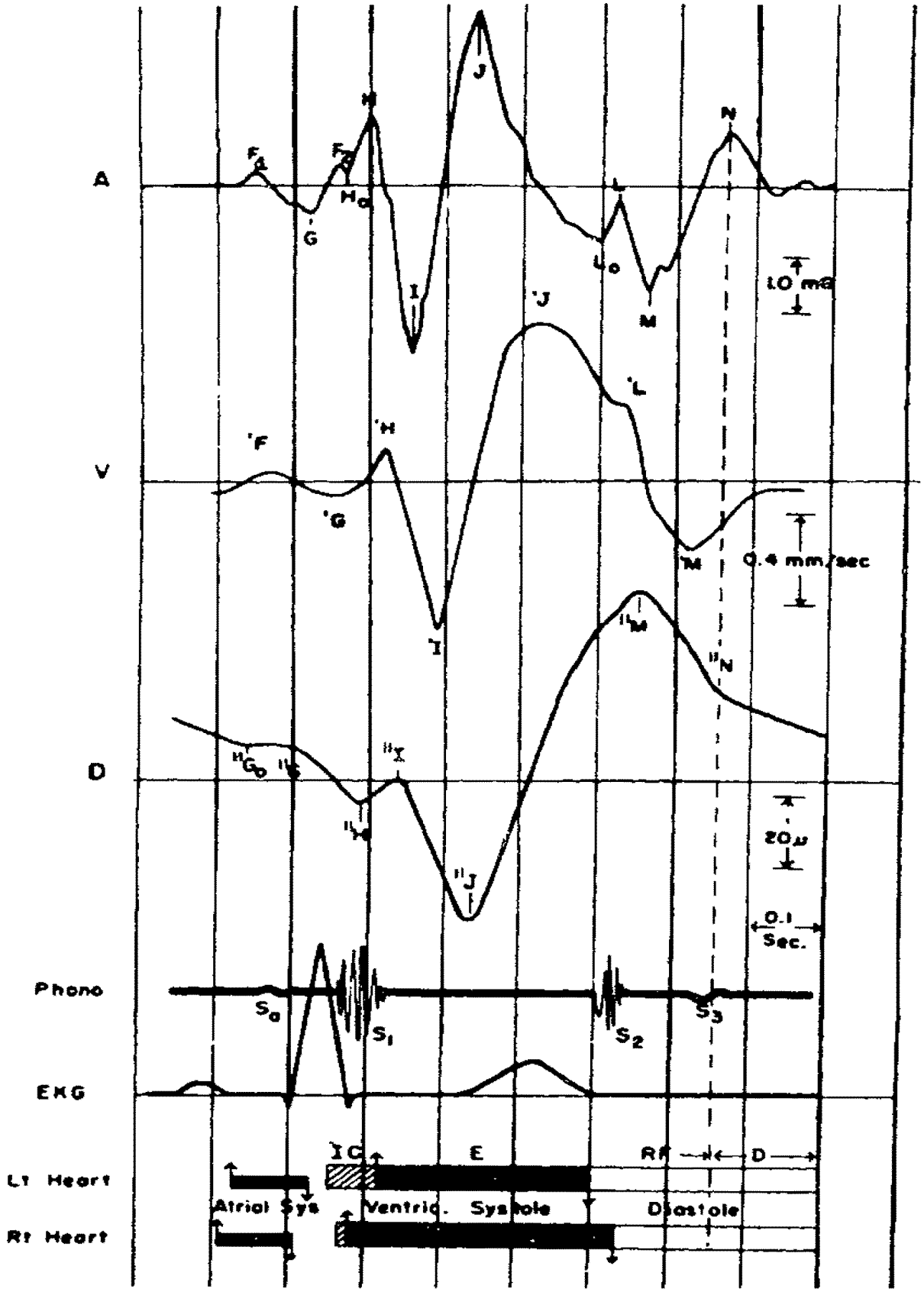


Fig. 1.—Average normal experimental ultra-low frequency displacement (D), velocity (V), and acceleration (A) ballistocardiograms. (From a paper by Scarborough et al., Am. J. Cardiol.[8]; courtesy of the authors and the publisher.)

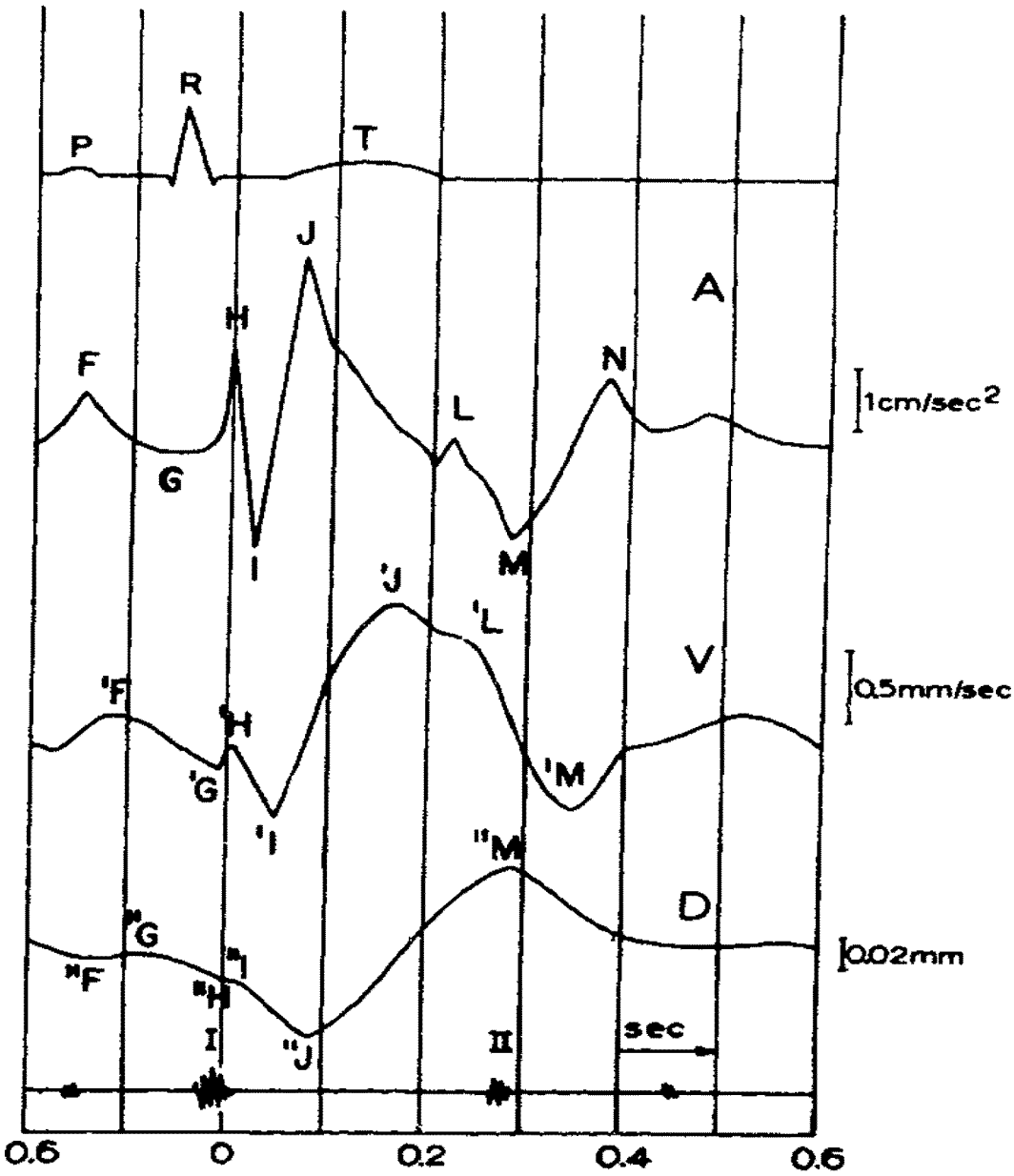


Fig. 1 (cont'd).—Composite drawing of the predicted ultra-low frequency ballistocardiograms. Other cardiac phenomena are given for easy time reference.

body frame. Discarding motion caused by breathing, the internal center of gravity of the subject changes almost periodically by the changing distribution of blood over the heart chambers and the large vessels.

Shortly after the onset of systole, when blood is driven first headward out of the heart to distend the great vessels, the internal center of gravity of blood, and thus of the whole body, moves headward in the body. Later, as the pulse wave spreads peripherally, blood accumulates at a greater distance from the heart in the more peripheral vessels, such that the center of gravity moves footward in the body. But if the body is free to move longitudinally, as it nearly is in the frequency range of interest when a ballistocardiograph of ultra-low natural frequency is used, the center of gravity of body, blood, and support does not move in space; it is the body that moves first footward and

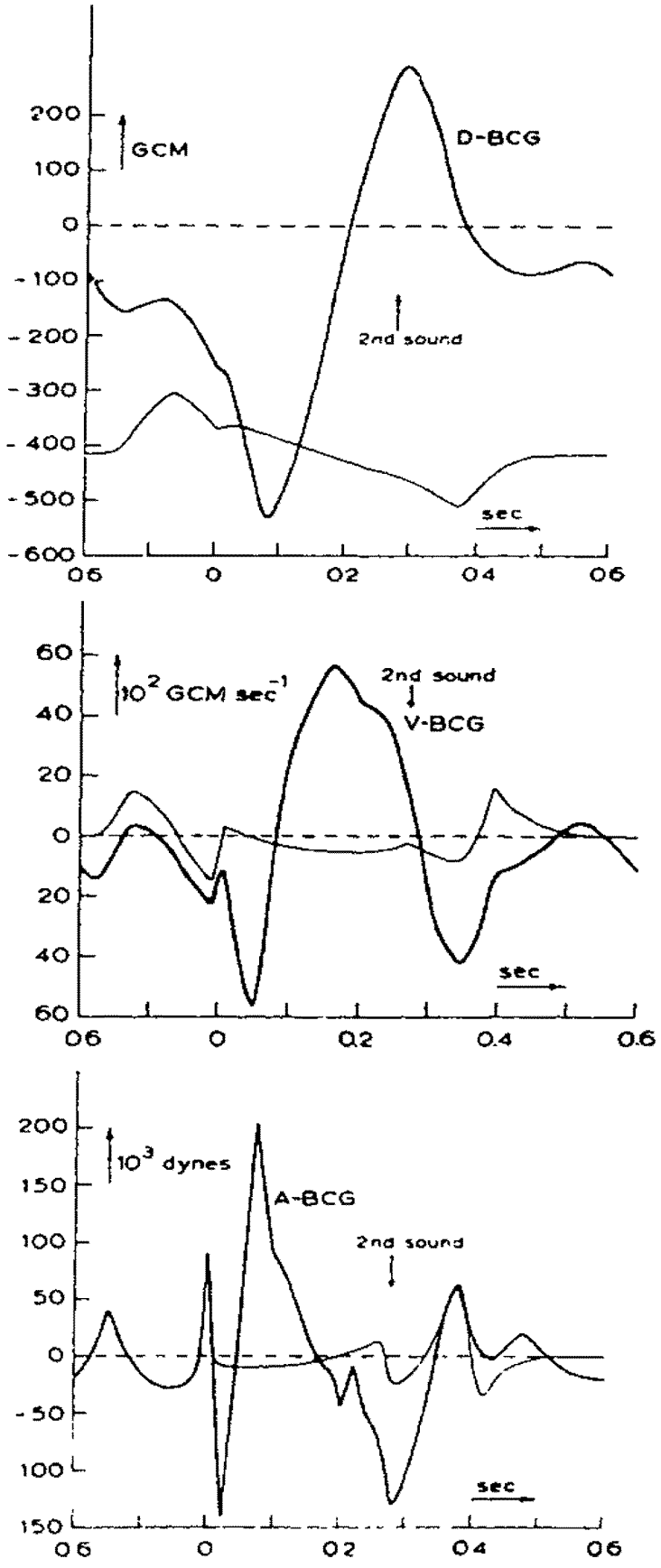


Fig. 2.—The heavy lines represent the computed total displacement (D-BCG), velocity (V-BCG), and acceleration (A-BCG) ballistocardiograms. The light lines are plots of the calculated contributions of the filling and emptying of the heart and of its motion.[11] Instant zero indicates the opening of the aortic and pulmonary valves. (Circulation, courtesy of the American Heart Association.)

then headward during the cardiac cycle. Average normal records of this motion are reproduced in the first part of figure 1.[8]

Obviously the distribution of blood changes within the body with each beat of the heart, but the changes in filling of the heart chambers, the arteries and the veins do not occur at the same time nor do they have the same sign. Even the change in filling of the arteries does not occur simultaneously because of the finite pulse wave velocity. In order to take time (phase) differences between distentions at various sites in the arterial trees into account, the larger arteries of both arterial trees were subdivided into small segments (115 in total). The mass of blood above the diastolic level of each segment—the excess mass—was then calculated as a function of time with intervals of 20 milliseconds for all of the 115 segments. Multiplication of these excess masses by the appropriate distance to a reference plane passed arbitrarily through the center of both ventricles and perpendicular to the longitudinal axis of the body and summation of the results, gives an estimate of the expected longitudinal displacement of the center of gravity as far as caused by the events inside the ventricle and the larger arteries.[9-10] Later, this work was expanded so as to include the contribution of cardiac filling, atrial contraction and the movement of the heart itself.[11] Figure 2 shows the final results for displacement, velocity, and acceleration, the latter two being calculated from the first. A compilation of the calculated ballistocardiograms is shown in the second part of figure 1. From these results it may be concluded that the change in distribution of blood in the heart and the large arteries accounts for the average normal experimental results.

Using the same method of approach it proved possible to calculate plethysmograms of both extremities and central sections of the body.[12] These results give a quantitative interpretation of the greater part of the well-known plethysmograms of the extremities.

Finally, another quantity of interest, namely the left ventricular ejection curve and thus the stroke volume, was calculated in absolute value from the same data. The computed stroke volume was 80 cc. and the ejection curve itself showed the early outflow peak and a small retrograde flow, which are found experimentally using modern flowmeters.[13,14]

These results were derived via lengthy calculations using slide rule and desk calculator. Nevertheless, they consititute only a single evaluation of a number of aspects of the average normal human circulatory system. To work out similar results in the same way for different situations, especially for abnormal conditions, is a rather unattractive, if not impossible task. However, the results, in quantitative terms, seem to be sufficiently promising to devise (a) a faster method of computing, (b) requiring fewer simplifications of the circulatory system and (c) including more quantities of interest, if possible.

Design of a Computer

The above described calculations could, of course, be programmed on a digital computer. This would certainly speed up the computations as desired,

but would, in the same form, not eliminate any of the simplifications that were introduced. The main simplification that should be removed at this stage is that the pulse pressures at the various sites in the arterial trees were taken from the literature instead of being determined only by the chosen situation of anatomy of the vasculature, by the elastic properties of the arteries and by the properties of the heart. To program this problem requires a very large digital computer, making the cost run high, and even then it is hard to account for non-linearities such as the presence of valves. (A program is presently being developed.[15]) A mathematical treatment of the events in the circulatory system taking into account the actual boundary conditions will probably be beyond the range of possibilities for at least some time to come, although such a treatment to solve special problems confined to parts of the system has been very fruitful.[16-19]

What is needed is obviously a physical analog rather than a mathematical one. Then the question arises whether one should prefer a hydrodynamic model to an electric one. Hydrodynamic models have been constructed[20,21] but meet with considerable technical difficulties since it is difficult to obtain tubes with a given modulus of elasticity, radius and wall-thickness. Moreover, it is not easy to adapt such a model to pre-set changes. On the other hand, hydrodynamic analogs have some features, such as the sleeve effect, which electrical models do not process automatically. On the basis of these considerations we decided to try to develop an electric analog computer, thereby introducing the possibility of adjustment as an attractive feature.

An electric analog that simulates the circulatory system should have electrical equivalents of the atria and ventricles and of the systemic and pulmonary trees. Each of these sections should be represented to a sufficient accuracy. An essential difficulty in such a set-up is the lack of information as to the manner of operation of the normal system. To mention a few points: Does a ventricle act as a pressure or as a flow source? To what extent are the elastic properties of the arterial walls nonlinear? Is the relationship between flow and pressure differences along the periphery a linear one? What is the relationship between pressure gradient and flow in central veins? We therefore decided to build the computer gradually, i.e., to remove simplifications and limitations one by one. It is hoped that such a procedure will aid in the quantitative evaluation of a variety of phenomena that are believed to occur in the circulatory system.

This paper will deal only with the derivation of an electric analog of the left ventricle and of the systemic arterial tree, both in a simple form and, for the present, without regulatory mechanisms.

Turning to the derivation of an electrical equivalent of the systemic arterial system, one must determine how such a tree of distensible tubes can be translated. This is possible by comparing pressure-flow relationships in a segment of distensible tubing with the telegraph equations which describe transmission of an electrical pulse through a uniform cable. The pressure drop per length in a tube with constant cross-sectional area and homogeneous and isotropic wall material is given by the (reduced) Navier-Stokes equation *(1a)*

$$-\frac{\delta p}{\delta x} = IW' + \frac{\rho}{S}\frac{\delta I}{\delta t} \qquad (1a)$$

where p = pressure,
I = flow
t = time
ρ = density of blood
S = πr^2 = cross-sectional area of the tube with internal radius r
W′ = resistance per length
x = axial coordinate.

The change in flow per length in such a segment of tubing is given by the continuity equation *(1b)*

$$-\frac{\delta I}{\delta x} = p\,G' + \frac{dS}{dp}\frac{\delta p}{\delta t} \qquad (1b)$$

where G′ = leakage per length, accounting for outflow through small branches

$\frac{dS}{dp}$ = distensibility of the tube.

The telegraph equations describe the change in voltage and in current per length in a uniform cable

$$-\frac{\delta V}{\delta x} = iR' + L'\frac{\delta i}{\delta t} \qquad (2a)$$

$$-\frac{\delta i}{\delta x} = \frac{V}{R_1'} + C'\frac{\delta V}{\delta t} \qquad (2b)$$

where V = voltage
i = current
R′ = resistance per length
L′ = inductance per length
C′ = capacitance per length
$1/R'_1$ = leakage per length
(the prime is used to denote quantities defined per length)

Equations *(1a)* and *(1b)* are analogous to equations *(2a)* and *(2b)* in both this and the reversed order. Thus, the equivalent pairs are *(1a)*–*(2a)* and *(1b)*–*(2b)* (mass-inductance analogy) or *(1a)*–*(2b)* and *(1b)*–*(2a)* (mass-capacitance analogy). For reasons of convenience we have chosen to use the mass-inductance analogy.[22] The hydrodynamic quantities are then "translated" into electrical quantities according to equations *(1a)*–*(2a)* and *(1b)*–*(2b)*.

pressure p	— voltage V
flow I	— current i
resistance per length W′	— resistance per length R′
inertance ρ/S	— inductance per length L′
distensibility dS/dp	— capacitance per length C′
leakage per length G′	— conductivity per length $1/R'_1$

An additional equivalent quantity is

volume ($=\int I dt$)	— charge Q ($=\int i dt$)

For the present it is assumed that inertial and viscous effects do not interact (no sleeve effect) (eq. (*1a*)). In addition it is assumed that Poiseuille's law holds. Then W′ can be written as

$$W' = \frac{8\pi\eta}{S^2} \tag{3}$$

where η = viscosity of blood.

We can express dS/dp in the anatomical and elastic properties of the tube[23] by

$$\frac{dS}{dp} = \frac{3S\,(a+1)^2}{E\,(2a+1)} \tag{4}$$

where E = Young modulus of the wall material that is assumed to be isotropic.

a = ratio between radius and wall-thickness.

The electrical analogy derived above can be constructed in different ways. In the first place, operational amplifiers could be used, and secondly, a passive line could be built. For reasons of simplicity and expenses we have chosen to construct a passive delay line for the arterial trees.[24] A segment of artery of length $\triangle$ x is then represented as in figure 3.[14]

For the actual construction a question that remains to be answered is what length of artery may be lumped as indicated in figure 3, without appreciably affecting the results, a point of discussion already raised by Landes.[25] Two extreme solutions are clearly inadequate. If one lumps an entire vascular tree, i.e., represents it by a single equivalent segment as in figure 3, then the pulse wave will lack a suitable electrical counterpart. The other extreme would be to use an infinite number of segments in series, as in figure 3. This, however, would preclude actual construction of such a model.

To find a practical compromise, a few studies were done of cases so chosen that the actual answers could be derived with the aid of formulas. These results could then be compared with the results obtained experimentally from the electrical circuitry.[23,24,26]

The amplitude of the "ballistocardiogram" was calculated as a function of frequency for the case of a single uniform artery with a homogeneous and isotropic wall with sinusoidal input flow at one end and closed at the other. The leakage through the wall was chosen as 5 liters per minute. The result written in equivalent electrical symbols reads

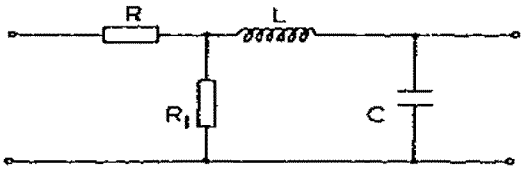


Fig. 3.—Electrical analog of a segment of artery of length $\triangle$x. The relation between R, L, R_1, and C in the figure and R′, L′, R_1′, and C′ in eq. (2) is: L = L′$\triangle$x, R = R′$\triangle$x, R_1 = R_1′/$\triangle$x, and C = C′$\triangle$x.

$$|x_c| = \frac{V_s C'\omega}{2\pi r^2 l\,(R'^2 + \omega^2 L'^2)^{1/4}\,(G'^2 + \omega^2 C'^2)^{3/4}} \left[\frac{e^{2\beta l} - 2e^{\beta l}\cos \alpha l + 1}{e^{2\beta l} + 2e^{\beta l}\cos \alpha l + 1}\right]^{1/2} \quad (5)$$

in which $|x_c|$ = amplitude of the displacement ballistocardiogram
V_s = stroke volume
$\omega = 2\pi f$
f = frequency

$$\alpha = \left[\frac{\omega^2 L'G' - R'G'}{2} + \tfrac{1}{2}[(R'^2 + \omega^2 L'^2)\,(G'^2 + \omega^2 C'^2)]^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

$$\beta = \left[\frac{R'G' - \omega^2 L'C'}{2} + \tfrac{1}{2}[(R'^2 + \omega^2 L'^2)\,(G'^2 + \omega^2 C'^2)]^{\frac{1}{2}}\right]^{\frac{1}{2}}$$

l = total length of tube representing an artery.

The numerical values for R', L', and C' were taken for a human thoracic aorta at a mean pressure of around 100 mm. Hg.[23,24,26]

$R' = 0.12$ g. cm.$^{-5}$ sec.$^{-1}$
$L' = 0.37$ g. cm.$^{-5}$
$C' = 12.5\ 10^{-6}$ g.$^{-1}$cc. sec.^{2}
$G' = 8.1\ 10^{-6}$ g.$^{-1}$cc. sec.
$l = 70$ cm.

Figure 4 shows the calculated amplitude of the displacement ballistocardiogram as a fully drawn line. Next the same quantity was secured experimentally from an electric model in which lumping was different in various experiments. The 70 cm. of artery was constructed as a single segment as in figure 3, as

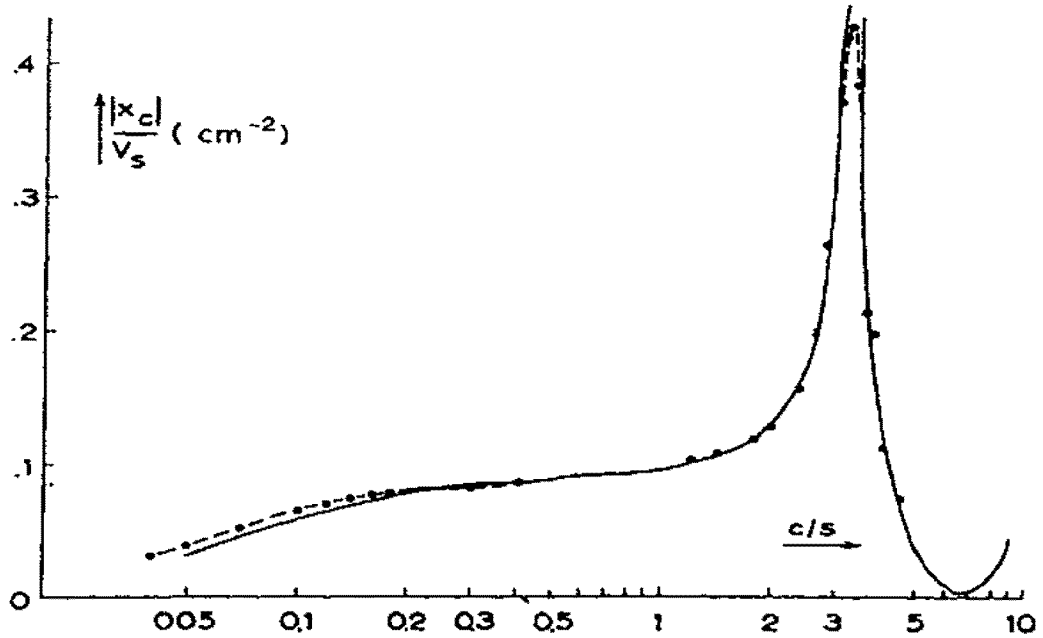


Fig. 4.—The ratio between the amplitude of the displacement ballistocardiogram ($|x_c|$) and the stroke volume (V_s) as a function of frequency for a thoracic aorta 70 cm. in length (theoretical: fully drawn; experimental from a delay line consisting of 20 segments in series: broken line).

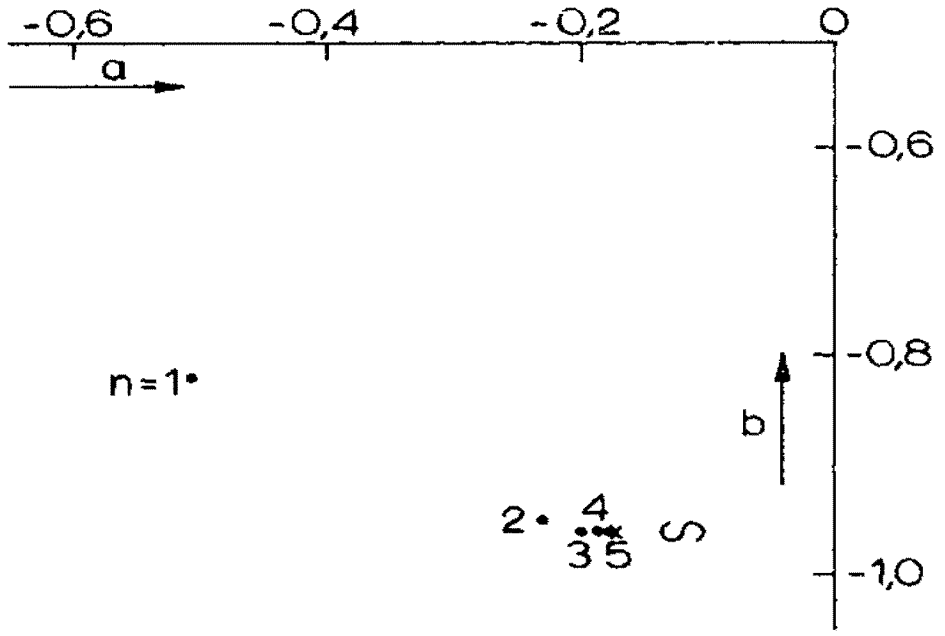


Fig. 5.—The ratio between the pressure amplitude 70 cm. away from the inlet of an infinitely long tube and the pressure amplitude at the inlet (V_{70}/V_0). The complex ratio V_{70}/V_0 is written as a + jb. The result calculated directly from the hemodynamical situation (infinite number of segments) is given by x. The results computed for the electrical analogy are plotted for n = 1 to 5.

two such segments, and as 5, 10, and 20 of such segments in series, in each case with different numerical values for R, L, C, and R_1 according to $R = R' \triangle x$, $L = L' \triangle x$, $C = C' \triangle x$, and $R_1 = R_1'/\triangle x$ in which $\triangle x$ is the length of vessel that is lumped. The results obtained in the case where 20 equal segments were used, i.e., 3.5 cm. of artery lumped, are also given in figure 4 (broken line). With such a fine subdivision there is excellent agreement except for the height of the resonance peak. In cases with less conspicuous resonance peaks, a considerably cruder subdivision may be adequate. A similar conclusion for a catheter was reached by Van Brummelen[27] when, using different constants, he found that lumping of 10 cm. gave results very close to the actual answer.

A different test was carried out on an infinitely long tube of the same size and elastic properties but without leakage ($G' = 0$). In the first place a calculation was made of the ratio between the pressure amplitude at a distance of 70 cm. from the inlet and the input pressure amplitude for a frequency of 2 cps. Next, with the aid of a digital computer, this same ratio was calculated in the electrical case for a variable degree of lumping. The length of the lumped segments was 70/n cm., with n = 1, 2, 3, 4, 5, 10, 20, etc., up to 100. The results have been plotted in figure 5 and show that a good agreement between the two types of results is already obtained when the segment length equals 17 cm.

Although the question as to what extent lumping is permitted will require further study, the conclusion seems justified that for a reasonable electrical analogy of the vascular trees, many electrical circuits should be used in series.

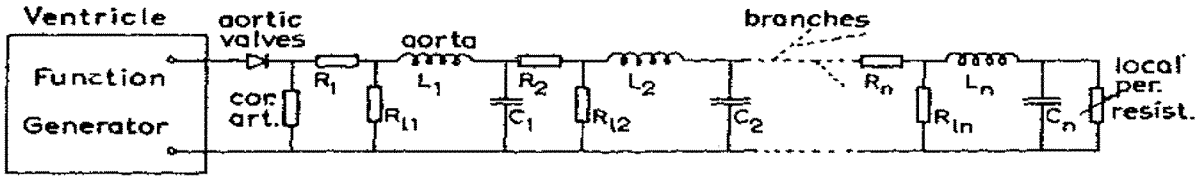


Fig. 6.—Outline of the electric model of the systemic circulatory system.

Adequate pulse transmission cannot be obtained by representing the systemic arterial tree by, e.g., 10 lumped segments. We have chosen segment lengths to be lumped in our delay line equal to a distance covered by the pulse wave in 10 milliseconds, a time short compared to the propagation time of the pulse wave. Thus, the length of a lumped segment is 6 cm. in an artery where the pulse wave velocity equals 6 m/sec. Sometimes it is even shorter where that happened to fit conveniently with the anatomy of the systemic arterial tree. This is in order to comply with both mathematical and anatomical requirements.

Set-up of Analog Computer

The computer in so far as it is complete at present is outlined in figure 6. The left ventricle is represented by a function generator, a periodic voltage source with an output impedance of less than 1 Ω. The output voltage of the function generator as a function of time can be adjusted at will by adding 20 step functions. For each step function the amplitude can be adjusted continuously between +30 Volt and −30 Volt. In addition, the moment at which each voltage step occurs can be chosen anywhere in the cycle. The sum of the 20 steps in voltage is fed into a filter to obtain a smooth output voltage. This voltage can easily be adjusted to simulate any desired ventricular pressure curve.

In addition to the possibility of shaping the function generator output, its repetition rate can be varied independently.

The delay line, representing the arterial tree, is built in accordance with the conclusions arrived at above. The numerical values of all elements are listed in table 1,[22,28] together with the anatomical data used in their calculation. The same value for the Young modulus of the wall material, viz. $E = 4.0\ 10^6$ g. cm.$^{-1}$ sec.$^{-2}$, was assumed throughout the systemic tree.[23] The operation frequency is a thousand times as high as the real one to get L and C values in a convenient range. The anatomy of the delay line itself is shown in figure 7.[28]

In between the function generator and the delay line, three diodes in parallel (type OY 5062) were placed so as to simulate the aortic valves.

The delay line is terminated by resistors representing the peripheral resistance. Their normal values were derived from the literature and are indicated in figure 7 (in k Ω).

From this computer one can record the following equivalent quantities: ventricular pressure, local arterial pressure, ventricular ejection flow, local arterial flow, and stroke volume. As yet only ventricular pressure, heart rate and local peripheral resistance are easily adjustable. Any other change has to be introduced by replacing one or more elements.

Table 1

D Indicates that the Segment Is Paired	Notation of Segment	Length of Segment (cm.)	Internal Radius (cm.)	Cross-sectional Area (cm.²)	Wall-thickness (cm.)		Ω	10^{-3} H	10^{-9} F
		$\triangle x$	r	S		n	R	L	C
	2a	2.0	1.44	6.51	0.158	9.11	0.0415	0.33	51.9
	2b	2.0	1.44	6.51	0.158	9.11	0.0415	0.33	51.9
	3a	2.0	1.17	4.30	0.134	8.73	0.096	0.49	33.2
	3b	3.9	1.17	4.30	0.134	8.73	0.185	0.96	64.4
	4a	5.2	0.98	3.02	0.117	8.40	0.50	1.83	58.5
	4b	5.2	0.98	3.02	0.117	8.40	0.50	1.83	58.5
	4c	5.2	0.98	3.02	0.117	8.40	0.50	1.83	58.5
	5a	5.3	0.85	2.27	0.105	8.05	0.91	2.48	43.2
	5b	5.3	0.85	2.27	0.105	8.05	0.91	2.48	43.2
	5c	5.3	0.85	2.27	0.105	8.05	0.91	2.48	43.2
D	6a	5.8	0.52	0.85	0.076	6.89	7.06	7.23	15.6
D	7a	5.8	0.41	0.53	0.066	6.21	18.2	11.5	8.92
D	7b	2.5	0.41	0.53	0.066	6.21	7.83	5.0	3.86
D	8a	6.3	0.29	0.26	0.056	5.18	82.0	25.0	4.13
D	8b	6.3	0.29	0.26	0.056	5.18	82.0	25.0	4.13
D	8c	1.9	0.29	0.26	0.056	5.18	24.7	7.75	1.24
D	9a	6.1	0.34	0.36	0.060	5.67	40.7	18.0	5.95
D	9b	6.1	0.34	0.36	0.060	5.67	40.7	18.0	5.95
D	9c	6.1	0.34	0.36	0.060	5.67	40.7	18.0	5.95
D	9d	6.1	0.34	0.36	0.060	5.67	40.7	18.0	5.95
D	9e	7.1	0.34	0.36	0.060	5.67	47.5	20.9	6.92
D	10a	6.3	0.28	0.25	0.054	5.19	88.7	26.7	3.99
D	10b	6.3	0.28	0.25	0.054	5.19	88.7	26.7	3.99
D	10c	6.3	0.28	0.25	0.054	5.19	88.7	26.7	3.99
D	11a	6.7	0.20	0.126	0.048	4.17	373.	55.9	1.87
D	11b	6.7	0.20	0.126	0.048	4.17	373.	55.9	1.87
D	11c	6.7	0.20	0.126	0.048	4.17	373.	55.9	1.87
D	11d	6.7	0.20	0.126	0.048	4.17	373.	55.9	1.87
D	11e	5.3	0.20	0.126	0.048	4.17	294.	44.2	1.48
D	12a	7.5	0.13	0.053	0.041	3.17	2350.	149.	0.715
D	12b	7.5	0.13	0.053	0.041	3.17	2350.	149.	0.715
D	12c	7.5	0.13	0.053	0.041	3.17	2350.	149.	0.715
D	12d	7.5	0.13	0.053	0.041	3.17	2350.	149.	0.715
D	12e	4.3	0.13	0.053	0.041	3.17	1350.	85.0	0.410
	13a	3.4	0.62	1.21	0.085	7.29	2.05	2.98	13.6
	14a	3.4	0.43	0.58	0.068	6.32	8.90	6.21	5.80
D	14b	6.8	0.43	0.58	0.068	6.32	17.6	12.4	11.6
D	15a	6.1	0.34	0.36	0.060	5.67	40.7	18.0	5.95
D	15b	5.6	0.34	0.36	0.060	5.67	37.4	16.5	5.46
D	16a	6.3	0.27	0.23	0.053	5.09	105.	29.0	3.61
D	16b	6.3	0.27	0.23	0.053	5.09	105.	29.0	3.61
D	16c	6.3	0.27	0.23	0.053	5.09	105.	29.0	3.61
D	16d	4.6	0.27	0.23	0.053	5.09	76.5	21.2	2.64
D	17a	6.7	0.20	0.13	0.048	4.17	373.	55.9	1.87
D	17b	6.7	0.20	0.13	0.048	4.17	373.	55.9	1.87
D	17c	6.7	0.20	0.13	0.048	4.17	373.	55.9	1.87
D	17d	3.7	0.20	0.13	0.048	4.17	205.	30.8	1.03
D	18a	7.1	0.16	0.08	0.043	3.72	976.	94.1	1.13
D	18b	7.1	0.16	0.08	0.043	3.72	976.	94.1	1.13
D	18c	7.1	0.16	0.08	0.043	3.72	976.	94.1	1.13
D	18d	2.2	0.16	0.08	0.043	3.72	303.	29.2	0.349
D	19a	7.9	0.11	0.04	0.040	2.75	4340.	218.	0.488
D	19b	4.3	0.11	0.04	0.040	2.75	2360.	119.	0.266
	20a	1.0	0.39	0.48	0.064	6.09	3.82	2.21	1.37
	21a	7.1	0.18	0.10	0.045	3.89	625.	75.3	1.45
	22a	6.3	0.27	0.23	0.053	5.09	105.	29.0	3.61
	23a	6.6	0.23	0.17	0.050	4.60	201.	41.2	2.58
D	24a	3.2	0.26	0.21	0.053	4.91	63.9	16.2	1.62
	25Aa	5.9	0.43	0.58	0.068	6.32	15.4	10.8	10.1
	25Ba	5.0	0.16	0.08	0.043	3.72	687.	66.3	0.792
	26Aa	5.9	0.39	0.48	0.064	6.09	22.5	13.0	8.10
	26Ab	5.9	0.39	0.48	0.064	6.09	22.5	13.0	8.10
	26Ac	5.9	0.39	0.48	0.064	6.09	22.5	13.0	8.10
	26Ad	3.1	0.39	0.48	0.064	6.09	11.8	6.85	4.26
	26Ba	5.9	0.39	0.48	0.064	6.09	22.5	13.0	8.10
	26Bb	5.9	0.39	0.48	0.064	6.09	22.5	13.0	8.10
	26Bc	5.9	0.39	0.48	0.064	6.09	22.5	13.0	8.10
D	27a	7.1	0.18	0.10	0.046	3.91	625.	74.6	1.46
D	27b	7.7	0.18	0.10	0.046	3.91	678.	80.8	1.58

In addition to the quantities that can be recorded, extensions were built to make possible recording of ballistocardiogram and plethysmograms. The method of deriving these quantities is given in figure 8.[25] Since the equivalent of the volume of blood contained by a segment of artery is given by the charge Q on the condensor in the equivalent electrical segment, the volume of an artery is, in the model, determined by the sum of the charges on the corresponding condensors. The change in volume as a function of time is thus given by the change in the sum of the charges as a function of time ($\triangle Q_i(t)$). Therefore, we may write for the plethysmogram amplitude:

$$\text{PlCG ampl} = \sum_i \triangle Q_i(t) \tag{6}$$

$$\text{Substituting } \triangle Q_i = C_i \triangle V_i \tag{7}$$

we find

$$\text{PlCG ampl} = \sum_i C_i \triangle V_i(t) \tag{8}$$

In other words, the amplitude of the plethysmogram is found from summation of the voltages ($V_i(t)$) on the condensors weighted with factors proportional to the values of the condensors.

For the circuitry of figure 8 it holds

$$V_{PlCG}(t) = k_p R_p \sum_i \frac{\triangle V_i(t)}{k_p R_{pi}} \tag{9}$$

in which

$$k_p R_{pi} = \frac{1}{C_i}, \tag{10}$$

and k_p is a constant still to be chosen.

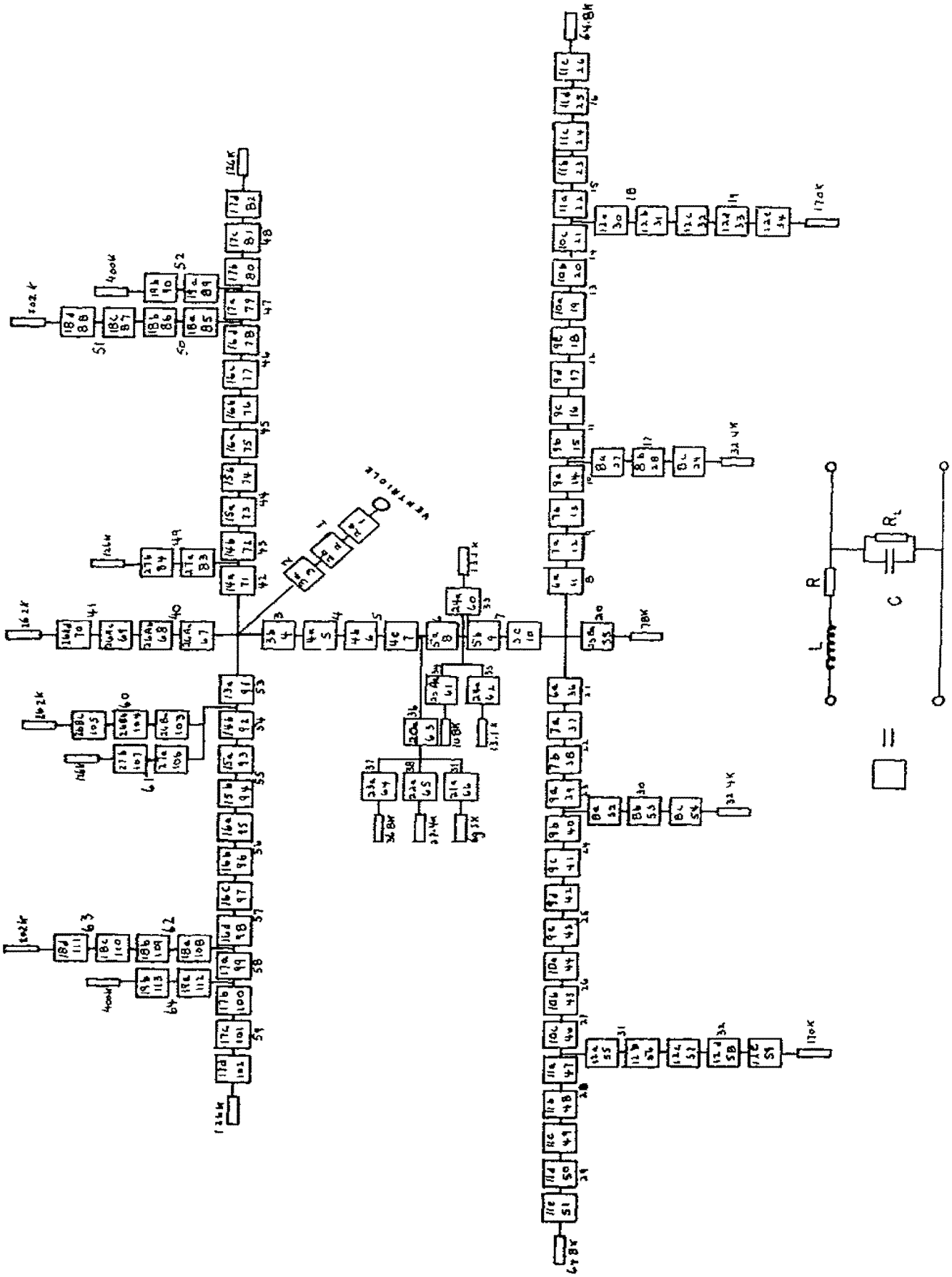
Equation (9) requires

$$\sum_i \frac{1}{R_{pi}} << \frac{1}{R_p} \tag{11}$$

Table I, legend. Numerical values of the elements of the delay line together with the anatomical data from which they were calculated. In the calculation of C the same value of E was used throughout: E = 4.0 10^6 g cm.$^{-1}$ sec.$^{-2}$. The segment notation is identical to that in figure 7. Consecutive segments of the same artery are distinguished by small letters following the number of the artery. Capitals following the number of an artery stand for superior or left (A) and inferior or right (B). The frequency multiplication factor 10^3 is accounted for in the columns giving L and C.

The key to the number of the arteries is:

2. aorta ascendens
3. arcus aortae
4. aorta thoracalis
5. aorta abdominalis
6. A. iliaca communis
7. A. iliaca externa
8. A. profundis femoris
9. A. femoralis
10. A. poplitea
11. A. tibialis posterior
12. A. tibialis anterior
13. A. anonyma
14. A. subclavia
15. A. axillaris
16. A. brachialis
17. A. ulnaris
18. A. radialis
19. A. interossea volaris
20. A. coelica
21. A. gastrica sinistra
22. A. lienalis
23. A. hepatica
24. A. renalis
25A. A. mesenterica sup.
25B. A. mesenterica inf.
26. A. carotis communis
27. A. vertebralis.


R
L
C
R_L

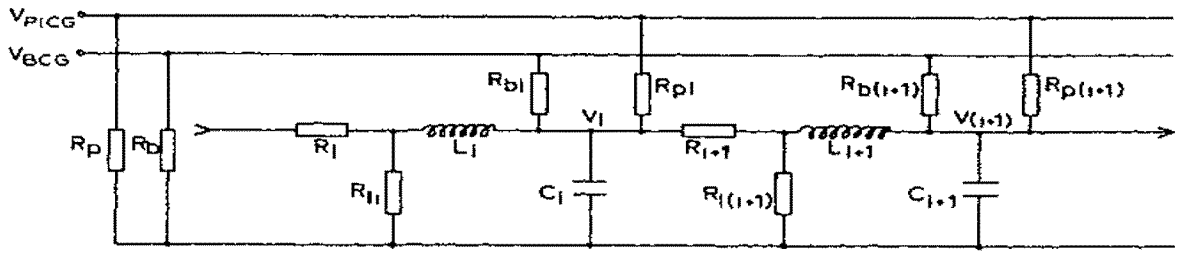


Fig. 8.—The method used to record plethysmogram and ballistocardiogram. The amplitude of the plethysmogram can be calculated from the measured voltage V_{PICG} using eq. (*9*), that of the displacement ballistocardiogram from the measured voltage V_{BCG} using eq. (*13*).

to be valid. This requirement, eq. (*10*), and the additional requirement that the capacitors may not be loaded appreciably by the resistors R_{pi}, make possible suitable choice of R_{pi}, k_p, and R_p. Since eqs. (*8*) and (*10*) are of the same form, the amplitude of the plethysmogram can easily be calculated from V_{PICG}.

In much the same way the amplitude of the displacement ballistocardiogram can be secured from the delay line. We find, applying eq. (*8*) and the definition of a center of gravity

$$\text{BCG ampl } (t) = \rho \sum_i y_i C_i \triangle V_i(t) \qquad (12)$$

where y_i = distance to a reference plane perpendicular to the longitudinal axis of the body.

From figure 8,

$$V_{BCG}(t) = k_b R_b \sum_i \frac{\triangle V_i(t)}{k_b R_{bi}} \qquad (13)$$

in which

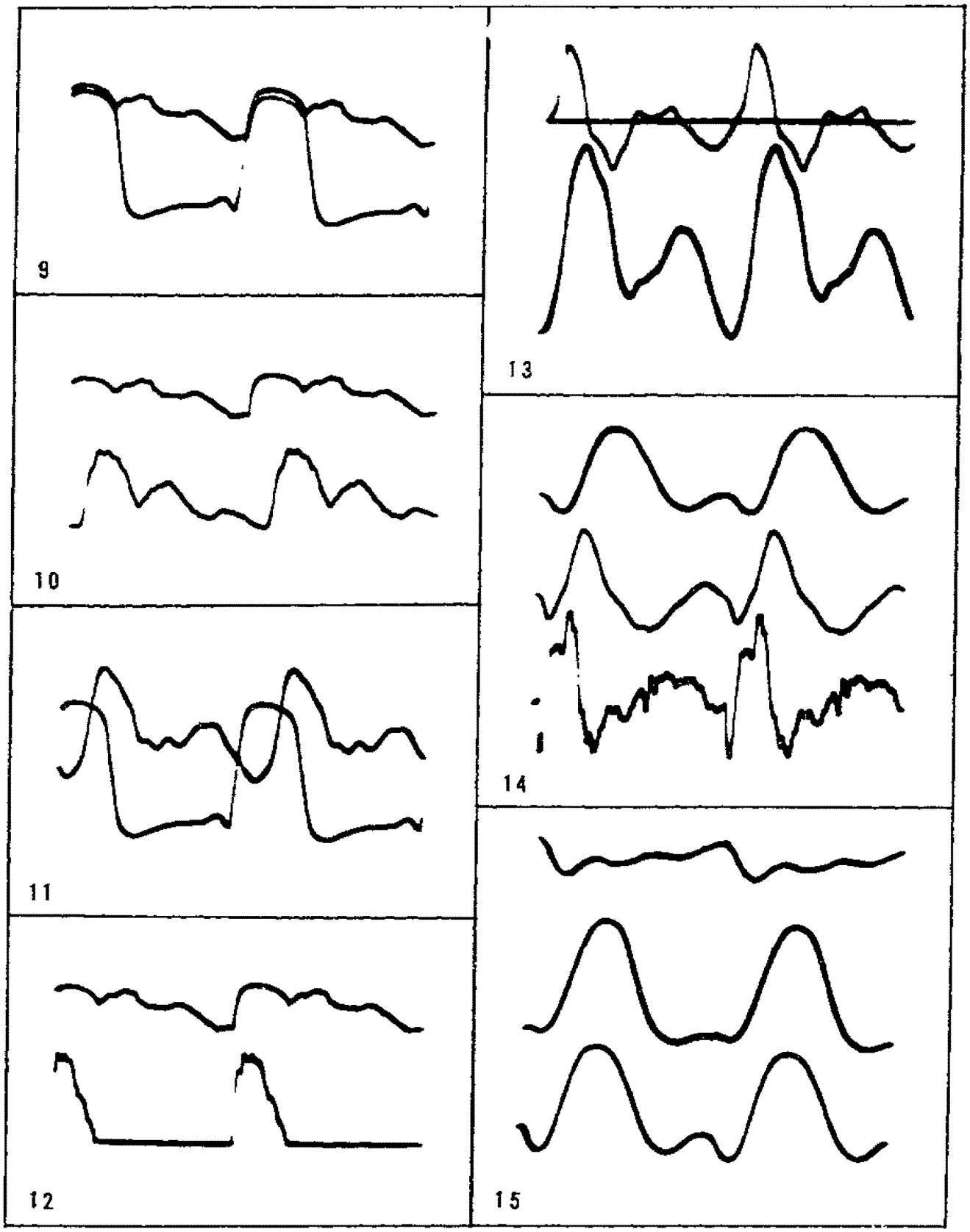
$$k_b R_{bi} = \frac{1}{\rho y_i C_i} \qquad (14)$$

From eq. (*14*) and analogous requirements as described for obtaining the plethysmogram, the values of R_{bi}, k_b, and R_b can be determined, thus making possible the calculation of the amplitude of the displacement ballistocardiogram from V_{BCG}.

The calculation of the hemodynamic quantities in absolute value from the electrical quantities may be done using the following conversions, in which a and b may be chosen arbitrarily.[28]

(electrical)		*(hemodynamic)*
b Volt	~	a mm. Hg
1 Ohm	~	1 g. cm.$^{-4}$ sec^{-1}

Fig. 7.—Anatomy of the delay line. Each box represents a segment of artery. The segment notation (top one in each box) is the same as in table 1. Peripheral resistances are denoted in kilo Ohms. The values of the leakage resistors R_l[11,22] are: in between the values and segment 2a: 32.4 kΩ (coronary arteries), in each of the segments 3b — 4c: 217 kΩ (intercostals). In all other segments there is no R_l. The calculated total leakage resistance equals 1630Ω.



Figs. 9–15

$$1 \text{ mA} \sim 1.33 \frac{a}{b} \text{ cc. sec}^{-1}$$

$$1 \text{ V}_{\text{PICG}} \sim \frac{1.33}{k_p R_p} \frac{a}{b} 10^6 \text{ cc.}$$

$$1 \text{ V}_{\text{BCG}} \sim \frac{1.33}{k_b R_b} \frac{a}{b} 10^6 \text{ g. cm.}$$

$$10^3 \text{ cps} \sim 1 \text{ cps}$$

Results

Some preliminary results obtained from the described computer are reproduced in figures 9–16. These results will be described briefly, using the equivalent hemodynamic nomenclature. A ventricular pressure pulse was chosen as an average normal one. All other quantities that can be recorded are then defined by this input function and the properties of the systemic vascular tree. Absolute values are given in the legends.

Figure 9 shows simultaneous tracings of the chosen ventricular pressure and of the pressure in the root of the aorta, while figures 10 and 11 show pressure recordings at different sites. The well-known physiologic phenomenon that the pulse pressure increases in the distal direction is also found in the computer, though in a somewhat overemphasized fashion. In addition, the pulse wave velocity in various parts of the systemic tree can be determined from these records. Simultaneous recordings of the pressure in the aortic root and the ejection flow are reproduced in figure 12. Retrograde flow is

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Fig. 9.—Simultaneous tracings of ventricular pressure (the steep upstroke runs from 0 to 120 mm. Hg) and pressure in the root of the aorta (109/73 mm. Hg). Heart rate in this and following figures 75/min.

Fig. 10.—Pressures in the root of the aorta (top) (109/73 mm. Hg) and at the distal end of the abdominal aorta (137/69 mm. Hg) (bottom) taken simultaneously. Note that the abdominal pressure pulse is delayed (pulse wave velocity 5.1 m/sec.) and shows a larger amplitude. Sensitivity the same for both records, lines for zero pressure are different.

Fig. 11.—Ventricular pressure (120/0 mm. Hg) together with the pressure pulse at the wrist (153/48 mm. Hg). The same line for zero pressure is valid for both records.

Fig. 12.—Simultaneous tracings of the pressure in the root of the aorta (109/73 mm. Hg) and the ventricular ejection curve. Peak flow: 630 cc./sec.; stroke volume 88 cc.

Fig. 13.—Top: Flow in a brachial artery. The horizontal line indicates zero flow. Positive peak amplitude 71 cc./sec. Bottom: Forearm plethysmogram. Max. peak-peak amplitude 1.6 cc.

Fig. 14.—From top to bottom: Displacement, velocity, and acceleration ballistocardiograms. Max. systolic peak-peak amplitudes 130 μ, 1.7 mm./sec., and 5.4 mg., respectively.

Fig. 15.—From top to bottom: Contribution to the displacement ballistocardiogram by all larger arteries caudal to the ventricle, distal to the ventricle, and caudal plus distal. Sensitivity same for the three tracings. Bottom curve is same as top curve in figure 14.

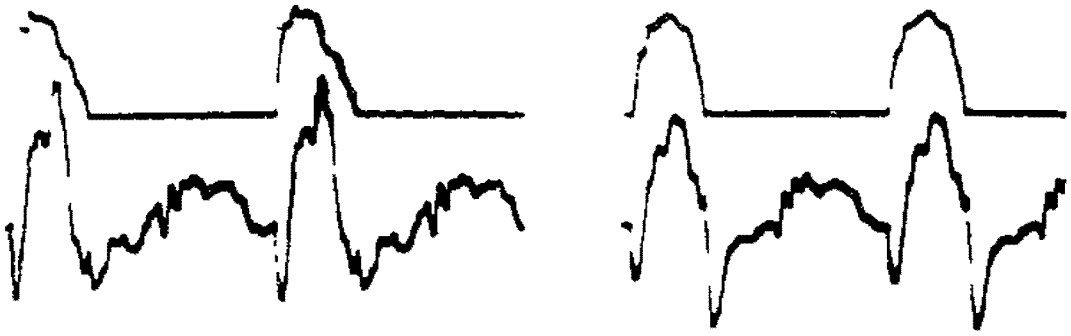

Fig. 16.—(Top): Left ventricular ejection curves and (bottom) acceleration ballistocardiograms in a "normal" case (left) with an early maximum in the flow curve and an "abnormal" case (right) with a later maximum in the flow curve. The amplitudes of the first negative and positive peak become smaller when ejection is slower, while the second negative peak increases in amplitude.[35]

zero, since the construction of the valves does not provide for the possibility of such flow.[29]

The oscillatory nature of the flow in the brachial artery is given in figure 13 (top), while the lower curve shows a forearm plethysmogram. Due to the fact that the computer produces peripheral pulse pressures of too large an amplitude, the periodic change in volume of a forearm as recorded from the computer is roughly twice that found in normal man.[12]

Figure 14 depicts (from top to bottom) the displacement, velocity, and acceleration ballistocardiograms as recorded from the computer which compare favorably, both with respect to amplitude and shape, to the ballistocardiograms in figure 1. An example of an experiment that can hardly be carried out in humans is given in figure 15, which shows how the displacement ballistocardiogram (bottom curve) is composed of contributions from the change in filling of the larger arteries headward (top) and footward (middle) of the left ventricle.

To what extent the acceleration ballistocardiogram is changed by a change in ventricular ejection pattern from maximum flow early in systole to maximum flow in the middle of systole, but with the same stroke volume, can be seen in figure 16. This result confirms Starr's findings in cadaver experiments, in which he simulated ventricular systole.[35] An essential difference between the experiments is that the simulated systoles in cadavers were nonperiodic, with long intervals in between, while the computer simulates periodic beats with a normal rate of 75/minute.

Discussion

The electrical model of the circulatory system described in this paper has fewer limitations than the mathematical model used previously to find a quantitative interpretation of the normal longitudinal ballistocardiogram. In the latter, all calculations were based upon local distensibility and average pressure curves taken from the literature. In the electrical analog—still limited to left ventricle and systemic arterial tree—only small segments of arteries are lumped so that there is an electrical equivalent of pulse wave velocity,

which is different from one place to another. This allows recording of pressure curves at a great many sites and eliminates the necessity of introducing pressure curves as independent information, as was required for the mathematical model.

A sufficient number of arteries has been included in the computer to simulate the widening of the arterial tree in the distal direction. The phenomenon that the computer overemphasizes the increase in pulse pressure in the distal direction has not yet been interpreted.

Several simplifications and assumptions had to be introduced. Solutions have been obtained to remove a few of these.[29] Two of them are: introduction of retrograde flow through the aortic valves, and representation of the effect of interaction between viscous and inertial effects (treated separately in eq. (*1a*) with different flow profiles). Further development will be required to remove the others.

A new model is under construction which, among other things, will allow easier adjustability of the properties of the analog to those of the arterial tree. This is necessary to make possible a more thorough study of the human circulatory system[30] covering physiologic as well as pathologic aspects, which is hardly possible without the use of models.[31-34]

Summary

A brief review is given of a mathematical model of the systemic arterial tree that was developed to find a quantitative interpretation of the human longitudinal ballistocardiogram. Derivation and description are presented of an electrical analog of the left ventricle and the systemic arterial tree that has fewer limitations than the mathematical model. Electrical equivalents of blood pressures, blood flows, vascular impedances, plethysmograms, and ballistocardiogram can be easily measured as a function of time, and in absolute value. Samples of such results are reproduced and compared with data reported in the literature.

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