THERMAL NOISE IN DOUBLE INJECTION DIODES OPERATING IN THE INSULATOR REGIME

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Abstract—Thermal noise calculations, based on the Langevin procedure, are given for a double injection diode operating in the insulator regime. The frequency range is restricted to frequencies smaller than the reciprocal free carrier transit times. It turns out that at high frequencies the spectral density of the ac short circuit current fluctuations can be described by Nyquist's formula. At low frequencies an additional term, proportional to $(1 + \omega^2\tau^2)^{-1}$ was found, where τ is the hole life time. The low frequency thermal noise level may be some orders of magnitude larger than the level at high frequencies. It is shown that in general the frequency dependent term will be masked by generation recombination noise. A special case is indicated where those contributions become comparable.

1. INTRODUCTION

SINCE noise data on double injection diodes operating in the insulator regime[1] become available[2], it is worthwhile to calculate the thermal noise in such devices. Current noise due to recombinations has been calculated before [4, 5].

We consider an insulator, provided with a hole-injecting (anode) and electron-injecting (cathode) contact. We will assume that the current is predominantly determined by carriers injected at the contacts and that the latter do not impose any constraint on current flow. In addition we assume that free carrier recombination takes place through localized Shockley–Read centres and that recombination of holes is monomolecular. The recombination mechanism involves the capture of a hole and subsequent recombination with an electron. The latter is supposed to be a very rapid process so that recombination is determined by the hole life time. Centres are neutral when electron occupied. Generations of free electrons and holes are ignored.

We have a planar geometry in mind, i.e. the contact spacing is small with respect to the cross-sectional dimensions of the diode so that a one dimensional treatment is warranted.

In order to calculate the current and voltage fluctuations, we avail ourselves of the Langevin procedure. This implies that the macroscopic equations describing particle current flow, and the

rate equations of electrons and holes are extended by so called Langevin source functions which formally describe the fluctuation processes involved.

Neglecting diffusion the equations for particle current flow, Poisson's Law, and the hole rate equation, for this case read:

$$J_p = q\mu_p pE + H_p(x, t) \quad (1)$$

$$J_n = q\mu_n nE + H_n(x, t) \quad (2)$$

$$J = J_p + J_n \quad (3)$$

$$\frac{\partial E}{\partial x} = \frac{q}{\epsilon\epsilon_0} (p - n) \quad (4)$$

$$\frac{\partial p}{\partial t} = -\frac{p}{\tau} - \frac{1}{q} \frac{\partial J_p}{\partial x} + H_{gr}(x, t) \quad (5)$$

where: J_p = hole current flow
 J_n = electron current flow
 n = free electron density
 p = free hole density
 E = electric field strength
 $\epsilon\epsilon_0$ = dielectric constant
 $-q$ = electronic charge
 $\mu_{n,p}$ = electron, hole mobility, assumed independent of E
 τ = free hole life time
 H_p = stochastic source function for thermal noise of holes

H_n = stochastic source function for thermal noise electrons

H_{gr} = stochastic source function associated with hole trapping.

Note that because of the assumptions made, we do not need the rate equation for free electrons, nor that for electrons in the recombination centres.

We will only consider frequencies small with respect to the reciprocal electron and hole transit times. This implies that displacement current can be ignored and hence

$$\frac{\partial J}{\partial x} = -\epsilon\epsilon_0 \frac{\partial^2 E}{\partial x \partial t} \approx 0. \quad (6)$$

The source functions H_p and H_n formally describe the random motion of electrons and holes induced by scattering processes [6]. For the cross correlation spectral densities S_{H_p} and S_{H_n} we write

$$S_{H_p}(x_1, x_2, f) = 4q^2 D_p p_0 \frac{\delta(x_1 - x_2)}{A} \quad (7)$$

$$S_{H_n}(x_1, x_2, f) = 4q^2 D_n n_0 \frac{\delta(x_1 - x_2)}{A} \quad (8)$$

where D_n and D_p are the diffusion constants for electrons and holes respectively, A is the cross-sectional area, δ is the Dirac delta function, and the subscript 0 indicates steady state values.

2. CALCULATIONS OF THERMAL NOISE

By eliminating n from equations (3) and (4) we can write for p

$$p = \frac{J - H_n - H_p + \epsilon\epsilon_0 \mu_n (\partial E / \partial x) E}{qE(\mu_n + \mu_p)}. \quad (9)$$

Substituting equation (1) into equation (5) and using equation (9) yields:

$$\begin{aligned} \frac{J - H_n - H_p}{E} + \tau \frac{\partial}{\partial t} \left(\frac{J - H_n - H_p}{E} \right) = \\ -\epsilon\epsilon_0 \mu_n \frac{\partial E}{\partial x} - b \frac{\partial}{\partial x} \left(E \frac{\partial E}{\partial x} \right) + \tau \left(\mu_p \frac{\partial H_n}{\partial x} - \mu_n \frac{\partial H_p}{\partial x} \right) + \\ + \tau q (\mu_n + \mu_p) H_{gr} \end{aligned} \quad (10)$$

where $b = \mu_n \mu_p \epsilon \epsilon_0 \tau$.

Following Friedonks [5], we will assume that for the diode operating in the insulator regime, the first term on the right hand side of equation (10) can be ignored. This implies that space charge is neglected.

Linearizing equation (10) by considering fluctuations around a steady state gives

$$J_0 = -bE_0 \frac{d}{dx} \left(E_0 \frac{dE_0}{dx} \right) \quad (11)$$

and

$$\begin{aligned} (\Delta J - H_n - H_p)E_0 - J_0 \Delta E + \tau \frac{\partial}{\partial t} [(\Delta J - H_n - H_p)E_0 - J_0 \Delta E] = \\ = -bE_0^2 \frac{\partial^2}{\partial x^2} (\Delta E E_0) + \tau E_0^2 \left[\mu_p \frac{\partial H_n}{\partial x} - \mu_n \frac{\partial H_p}{\partial x} + q(\mu_n + \mu_p) H_{gr} \right] \end{aligned} \quad (12)$$

The use of equations (7) and (8) can be justified by the fact that in the ohmic regime this formula leads to the familiar Nyquist expression for thermal noise if the Einstein relations are valid, i.e. if $qD_n = \mu_n kT$ and $qD_p = \mu_p kT$, where k is Boltzmann's constant and T is the device temperature. Also in our case we will assume that the Einstein relations hold. This implies that the velocity distribution is maxwellian, despite the applied electric field.

In the Langevin source functions electrons and holes are treated as independent. Correlation in their densities is taken into account by using the equations (1)–(5).

where steady state values are denoted by subscript 0. Making a Fourier analysis of equation (12), considering truncated time samples of length T , one finds

$$\begin{aligned} -bE_0^2 \frac{\partial^2}{\partial x^2} (\tilde{E} E_0) + J_0 (1 + i\omega\tau) \tilde{E} = \\ E_0 (1 + i\omega\tau) (\tilde{J} - \tilde{H}_n - \tilde{H}_p) - \\ - \tau E_0^2 \left[\mu_p \frac{\partial \tilde{H}_n}{\partial x} - \mu_n \frac{\partial \tilde{H}_p}{\partial x} + q(\mu_n + \mu_p) \tilde{H}_{gr} \right] \end{aligned} \quad (13)$$

where Fourier transforms of corresponding

quantities are denoted by tilde, i is the imaginary unit, and ω the circular frequency. Upon integration of equation (13) over x using partial integrations for the first term on the left hand side one finds

$$\begin{aligned} & \int_0^L 2b\tilde{E}E_0 \frac{d}{dx} \left(E_0 \frac{dE_0}{dx} \right) dx + J_0(1+i\omega\tau)\tilde{V} \\ &= V_0(1+i\omega\tau)\tilde{J} + \int_0^L (1+i\omega\tau)E_0(\tilde{H}_n + \tilde{H}_p) dx + \\ &+ \int_0^L \tau E_0^2 \left[\mu_p \frac{\partial \tilde{H}_n}{\partial x} - \mu_n \frac{\partial \tilde{H}_p}{\partial x} + q(\mu_n + \mu_p)\tilde{H}_{gr} \right] dx \end{aligned} \quad (14)$$

where the usual boundary conditions $E(0) = E(L) = 0$ are used, L = contact spacing, V_0 = d.c. applied voltage, and $\tilde{V}$ = Fourier transform of the a.c. voltage. Using equation (11) in rewriting the first term on the left hand side yields after some algebra:

$$\begin{aligned} \tilde{V} &= \frac{1}{J_0(3+i\omega\tau)} \left\{ V_0(1+i\omega\tau)\tilde{J} \right. \\ &+ \int_0^L (1+i\omega\tau)E_0(\tilde{H}_n + \tilde{H}_p) dx + \\ &+ \left. \int_0^L \tau E_0^2 \left[\mu_p \frac{\partial \tilde{H}_n}{\partial x} - \mu_n \frac{\partial \tilde{H}_p}{\partial x} + q(\mu_n + \mu_p)\tilde{H}_{gr} \right] dx \right\}. \end{aligned} \quad (15)$$

Putting $\tilde{H}_n$, $\tilde{H}_p$ and $\tilde{H}_{gr}$ equal to zero in equation (15) one finds the already known result[4, 5] for the a.c. small signal admittance

$$Y \equiv \frac{-A\tilde{J}}{\tilde{V}} = \frac{I_0}{V_0} \frac{3+i\omega\tau}{1+i\omega\tau} \quad (16)$$

where I_0 = d.c. current. The J_0 , V_0 characteristic can be calculated from equation (11) and is well known to be[1]

$$J_0 = -\frac{125}{18} \epsilon \epsilon_0 \mu_n \mu_p \tau \frac{V_0^3}{L^5}. \quad (17)$$

Turning to the noise we put $\tilde{J} = 0$ in equation (15). The noise sources H_n , H_p and H_{gr} are as-

sumed to be statistically independent. The resulting noise contributions in voltage or current induced by the sources mentioned can be calculated separately. Since the g - r noise has been calculated before[4, 5], we will omit $\tilde{H}_{gr}$ in equation (15) and only calculate the thermal noise. For the open circuit voltage fluctuations we thus find

$$\begin{aligned} \tilde{V} &= \frac{1}{J_0(3+i\omega\tau)} \int_0^L \left[(1+i\omega\tau)E_0(\tilde{H}_n + \tilde{H}_p) + \tau E_0^2 \right. \\ &\quad \left. \left(\mu_p \frac{\partial \tilde{H}_n}{\partial x} - \mu_n \frac{\partial \tilde{H}_p}{\partial x} \right) \right] dx. \end{aligned} \quad (18)$$

The spectral density S_V can be found from

$$S_V(f) = \lim_{T \rightarrow \infty} \frac{2}{T} \langle \tilde{V} \tilde{V}^* \rangle \quad (19)$$

using

$$S_{H_n}(x_1, x_2, f) = \lim_{T \rightarrow \infty} \frac{2}{T} \langle \tilde{H}_n(x_1) \tilde{H}_n^*(x_2) \rangle \quad (20)$$

$$S_{H_p}(x_1, x_2, f) = \lim_{T \rightarrow \infty} \frac{2}{T} \langle \tilde{H}_p(x_1) \tilde{H}_p^*(x_2) \rangle \quad (21)$$

$$\lim_{T \rightarrow \infty} \frac{2}{T} \langle \tilde{H}_n(x_1) \tilde{H}_p^*(x_2) \rangle = 0 \quad (22)$$

where the asterisk denotes complex conjugate quantities. From equations (18)–(22) we find for S_V

$$\begin{aligned} S_V(f) &= \frac{1}{J_0^2(9+\omega^2\tau^2)} \int_0^L \int_0^L dx_1 dx_2 \left[(1+\omega^2\tau^2)E_0(x_1)E_0(x_2)S_{H_n}(x_1, x_2, f) + \right. \\ &\quad + (1+\omega^2\tau^2)E_0(x_1)E_0(x_2)S_{H_p}(x_1, x_2, f) + \\ &\quad + (1+i\omega\tau)\tau\mu_p E_0(x_1)E_0^2(x_2) \frac{\partial}{\partial x_2} S_{H_n}(x_1, x_2, f) - \\ &\quad - (1+i\omega\tau)\tau\mu_n E_0(x_1)E_0^2(x_2) \frac{\partial}{\partial x_2} S_{H_p}(x_1, x_2, f) + \\ &\quad \left. + (1-i\omega\tau)\tau\mu_p E_0^2(x_1)E_0(x_2) \frac{\partial}{\partial x_1} S_{H_n}(x_1, x_2, f) + \right. \\ &\quad \left. + (1-i\omega\tau)\tau\mu_n E_0^2(x_1)E_0(x_2) \frac{\partial}{\partial x_1} S_{H_p}(x_1, x_2, f) \right] \end{aligned}$$

$$\begin{aligned}
& + \tau^2 \mu_p^2 E_0^2(x_1) E_0^2(x_2) \frac{\partial^2}{\partial x_1 \partial x_2} S_{H_n}(x_1, x_2, f) - \\
& - (1 - i\omega\tau) \tau \mu_n E_0^2(x_1) E_0(x_2) \frac{\partial}{\partial x_1} S_{H_p}(x_1, x_2, f) \\
& + \tau^2 \mu_n^2 E_0^2(x_1) E_0^2(x_2) \frac{\partial^2}{\partial x_1 \partial x_2} S_{H_p}(x_1, x_2, f) \Big]. \quad (23)
\end{aligned}$$

Rewriting equations (7) and (8) using the Einstein relations and putting

$$n_0 \approx p_0 = \frac{J_0}{q(\mu_n + \mu_p)} \frac{1}{E_0} \quad (24)$$

yields

$$S_{H_n} = 4kT \frac{J_0}{A} \frac{\mu_n}{\mu_n + \mu_p} \frac{\delta(x_1 - x_2)}{E_0(x_1)} \quad (25)$$

and

$$S_{H_p} = 4kT \frac{J_0}{A} \frac{\mu_p}{\mu_n + \mu_p} \frac{\delta(x_1 - x_2)}{E_0(x_1)}. \quad (26)$$

Using equations (25) and (26) in equation (23) and evaluating the integrals, the 3rd, 4th, 5th and 7th terms turn out to be zero. We finally find with the help of equation (11)

$$S_V = 4kT \frac{1 + \omega^2 \tau^2}{9 + \omega^2 \tau^2} \left[\frac{V_0}{I_0} + \frac{\tau}{1 + \omega^2 \tau^2} \frac{4L}{\epsilon \epsilon_0 A} \right]. \quad (27)$$

For the short circuit current spectral density $S_I = S_V |Y|^2$ we obtain

$$S_I = 4kT \frac{I_0}{V_0} \left[1 + \frac{\tau}{(1 + \omega^2 \tau^2)} \frac{I_0 4L}{V_0 \epsilon \epsilon_0 A} \right]. \quad (28)$$

3. DISCUSSION

At high frequencies only the first term on the right hand side of equation (28) is important and equation (28) reduces to the familiar Nyquist expression, since from equation (16) we see that for $\omega \gg 1/\tau$

$$\text{Re}(Y) = \frac{I_0}{V_0}. \quad (28a)$$

It should be noted that the high frequency result can be obtained directly from the current equation. Since for frequencies larger than the reciprocal free carrier common lifetime, fluctuations in the

free carrier densities may be neglected, one has

$$\frac{\Delta J}{\sigma(x)} = \Delta E(x) + \frac{H_n + H_p}{\sigma(x)} \quad (28b)$$

where $\sigma(x)$ is the steady state conductivity. Making a Fourier analysis and integrating over x yields

$$\tilde{J}_A \frac{V_0}{I_0} = -\tilde{V} + \int_0^L \frac{\tilde{H}_n + \tilde{H}_p}{\sigma} dx. \quad (28c)$$

Considering short circuit current fluctuations we have

$$\tilde{I} = -\frac{I_0}{V_0} \int_0^L \frac{\tilde{H}_n + \tilde{H}_p}{\sigma} dx \quad (28d)$$

and

$$S_I = \frac{I_0^2}{V_0^2} \int_0^L \int_0^L \frac{S_{H_n}(x_1, x_2, f) + S_{H_p}(x_1, x_2, f)}{\sigma(x_1)\sigma(x_2)} dx_1 dx_2. \quad (28e)$$

Using equations (25) and (26) we find

$$S_{H_n} + S_{H_p} = 4kT \sigma(x_1) \frac{\delta(x_1 - x_2)}{A}. \quad (28f)$$

Substituting this result in equation (28e) we find

$$S_I = 4kT \frac{I_0}{V_0}. \quad (28g)$$

Note that in deriving equation (28g) no specific assumptions concerning carrier injection have been made and therefore the result seems quite general. Experimental evidence in support of this result for the semiconductor regime is given by Lee and Nicolet [3].

Note also that by putting $\tilde{H}_n = \tilde{H}_p = 0$ in equation (28c) directly follows

$$Y = \frac{I_0}{V_0}. \quad (28h)$$

The second term on the right hand side of equation (28) is important only at low frequencies. By introducing $R_0 = V_0/I_0$ and $C_0 = \epsilon \epsilon_0 A/L$ one can

rewrite equation (28) in the following form

$$S_I = \frac{4kT}{R_0} \left[1 + \frac{4}{1 + \omega^2 \tau^2} \cdot \frac{\tau}{R_0 C_0} \right] \quad (28i)$$

this form is somewhat more revealing than equation (28) in that C_0 is clearly the geometrical capacitance and $R_0 C_0$ is related to the ohmic or dielectric relaxation time. Moreover it shows clearly that the second term between brackets can be several orders of magnitude larger than one.

Since the thermal noise source enters rate equation (5) through the divergence of the current, one expects a frequency dependent noise contribution of the generation-recombination type. Therefore it is surprising that such a term is absent in the ohmic regime. This becomes clear however, by inspection of equation (23). The frequency dependent contributions stem here from the 6th and 8th term. Now it can be shown that these contributions are nonexistent when the steady state electric field strength E_0 is space independent. This explains mathematically the difference in outcome for the ohmic and insulator regime. Apparently the fact that current is predominantly recombination limited in the insulator regime has something to do with it. The random thermal movement of carriers, the origin of thermal noise, gives rise to space charge fluctuations, which in the ohmic regime are relaxed through ohmic conduction only, whereas in the insulator regime this occurs partly through the much slower recombination process.

In general the frequency dependent thermal noise contribution will be masked by $g-r$ noise as we will show. Driedonks found for the short circuit current spectral density due to recombinations in a double injection diode operating in the insulator regime

$$S_{I_{gr}} = \frac{50}{21} \frac{q(\mu_n + \mu_p)}{L^2} I_0 V_0 \frac{\tau}{1 + \omega^2 \tau^2}. \quad (29)$$

Adding the right hand sides of equations (28) and (29) we find for the total spectral density

$$S_{I_{tot}} = 4kT \frac{I_0}{V_0} + \frac{\tau}{1 + \omega^2 \tau^2} (C_{th} + C_{gr}) \quad (30)$$

where

$$C_{th} = \frac{I_0^2 16kTL}{V_0^2 \epsilon \epsilon_0 A} \text{ and } C_{gr} = \frac{50q(\mu_n + \mu_p)I_0 V_0}{21L^2}$$

So

$$\frac{C_{th}}{C_{gr}} = \frac{168I_0 kTL^3}{25V_0^3 q(\mu_n + \mu_p) \epsilon \epsilon_0 A}. \quad (31)$$

With the help of equation (17) this reduces to

$$\frac{C_{th}}{C_{gr}} = \frac{70}{3} \frac{L_a^2}{L^2} \quad (32)$$

where

$$L_a = \left[\frac{2kT\mu_n\mu_p\tau}{q(\mu_n + \mu_p)} \right]^{1/2}$$

is the ambipolar diffusion length. A necessary condition for neglecting diffusion and the validity of our calculations is that the device length is much larger than the ambipolar diffusion length [1]. Although a special case where $C_{th} \approx C_{gr}$ is feasible because of the numerical factor in equation (32), one is tempted to state that mostly $C_{th} \ll C_{gr}$. It would be interesting, however, to check experimentally whether the condition $(L/L_a) < 5$, which would imply $C_{th} > C_{gr}$, is still consistent with the model chosen.

Bilger *et al.* [2] leave no doubt that their device is operating in the insulator regime. The ratio L/L_a is approximately 3 in their case. This implies according to equation (32) that $C_{th}/C_{gr} \approx 2.5$. Unfortunately, however, no clear low frequency plateaus are found so that no unambiguous interpretation in terms of the theory presented in this paper is possible. If one follows the procedure of Bilger *et al.* by fitting $(1 + \omega^2 \tau^2)^{-1}$ curves to the experimental spectra, assuming τ independent of the injection level, low frequency plateaus are found in the insulator regime which are in fair agreement to our equation (30), i.e. these are to a large extent due to thermal noise.

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