LINE WIDTH IN MAGNETIC RESONANCE DUE TO EMISSION OF RADIATION

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Line width in ferromagnetic resonance is found to consist of the magnon scattering, the superradiance, and a new effect due to the simultaneous presence of magnon scattering and the superradiance.

We wish to present the line width of a uniform magnon in ferromagnetic resonance arising from the superradiance and a new process arising from the non-commutation between the superradiance and the magnon-magnon surface scattering.

Consider a one-magnon-one-photon interaction [1] arising from the x -component of the magnetic vector of the electromagnetic field and the magnetization, $g\beta\hbar_x \cdot S_x$, and a magnon-magnon surface pit scattering [2], so that the Hamiltonian of the system assumes the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2 \quad (1)$$

$$\mathcal{H}_0 = \hbar\omega_0 b_0^\dagger b_0 + \hbar\Omega_0 a^\dagger a_0 + \sum_k \hbar\Omega_k a_k^\dagger a_k \quad (2)$$

$$\mathcal{H}_1 = i\alpha(b_0^\dagger - b_0)(a_0 + a_0^\dagger) \quad (3)$$

$$\mathcal{H}_2 = \sum_k F_k(a_0^\dagger a_k + a_0 a_k^\dagger) \quad (4)$$

where the coupling constant α in the free space is $(\pi\hbar M_0 V\hbar\omega_0)^{1/2}$ whereas, in the wave guide,

$$\alpha = [3\hbar M_0 V\hbar\omega_0 \lambda^3 / (2ab\lambda_g)]^{1/2} \cos k_g l \quad (5)$$

where M_0 is the magnetization, V is the volume of the sample, ω_0 the unperturbed photon frequency, λ the wave length in the free space and λ_g in the guide and l are the distance of the sample from the terminating end of the guide of area of cross section ab ; F_k is the structure factor in the coupling between the uniform magnon and finite wave vector magnon at the surface of the sample inside a dipolar spread band.

$a_{0,k}^\dagger$ and $a_{0,k}$ are the magnon creation and destruc-

tion operators while $b_0^\dagger$ and b_0 are the photon creation and annihilation operators.

We calculate the equation of motion of the Green function of the uniform magnon and the related one-particle Green functions which after taking the Fourier transform are found to be

$$(E - \hbar\Omega_0) \langle\langle a_0 | a_0^\dagger \rangle\rangle = \frac{\hbar}{2\pi} - i\alpha \langle\langle b_0 | a_0^\dagger \rangle\rangle \quad (6a)$$

$$+ i\alpha \langle\langle b_0^\dagger | a_0^\dagger \rangle\rangle + \sum_k F_k \langle\langle a_k | a_0^\dagger \rangle\rangle$$

$$(E - \hbar\omega_0) \langle\langle b_0 | a_0^\dagger \rangle\rangle = i\alpha \langle\langle a_0 | a_0^\dagger \rangle\rangle + i\alpha \langle\langle a_0^\dagger | a_0^\dagger \rangle\rangle \quad (6b)$$

$$(E + \hbar\omega_0) \langle\langle b_0^\dagger | a_0^\dagger \rangle\rangle = i\alpha \langle\langle a_0^\dagger | a_0^\dagger \rangle\rangle + i\alpha \langle\langle a_0 | a_0^\dagger \rangle\rangle \quad (6c)$$

$$(E - \hbar\Omega_k) \langle\langle a_k | a_0^\dagger \rangle\rangle = \sum_k F_k \langle\langle a_0 | a_0^\dagger \rangle\rangle \quad (6d)$$

$$(E + \hbar\Omega_k) \langle\langle a_k^\dagger | a_0^\dagger \rangle\rangle = - \sum_k F_k \langle\langle a_0^\dagger | a_0^\dagger \rangle\rangle \quad (6e)$$

$$(E - \hbar\Omega_0) \langle\langle a_0^\dagger | a_0^\dagger \rangle\rangle = - i\alpha \langle\langle b_0^\dagger | a_0^\dagger \rangle\rangle \quad (6f)$$

$$+ i\alpha \langle\langle b_0 | a_0^\dagger \rangle\rangle - \sum_k F_k \langle\langle a_k^\dagger | a_0^\dagger \rangle\rangle$$

Simultaneous solution of above gives the one-uniform magnon Green function as

$$(E - \hbar\Omega_0 - \Sigma) \langle\langle a_0 | a_0^\dagger \rangle\rangle = \hbar/2\pi \quad (7)$$

with the self-energy of the uniform magnon given by

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$$\begin{aligned} \Sigma = & \sum_k' \frac{F_k^2}{E - \hbar\Omega_k} - \alpha^2 \left(\frac{1}{E + \hbar\omega_0} - \frac{1}{E - \hbar\omega_0} \right) \\ & - \alpha^4 \left(\frac{1}{E + \hbar\omega_0} - \frac{1}{E - \hbar\omega_0} \right)^2 \\ & \times \left[E + \hbar\Omega_0 - \sum_k' \frac{F_k^2}{E + \hbar\Omega_k} \right. \\ & \left. - \alpha^2 \left(\frac{1}{E + \hbar\omega_0} - \frac{1}{E - \hbar\omega_0} \right) \right]^{-1}. \end{aligned} \quad (8)$$

The life time of the uniform magnon is determined by $1/\tau = -(2/\hbar)\text{Im } \Sigma$, where the imaginary part of the self energy is defined by Dirac's identity, $\lim_{\epsilon \rightarrow 0} 1/(x \pm i\epsilon) = (P/x) \mp i\pi\delta(x)$. The dominant contribution to the line width is then found to be $1/\tau = \Delta H = \Delta H_1 + \Delta H_2 + \Delta H_3$ (9)

where

$$\Delta H_1 = \frac{2\pi}{\hbar} \sum_k' F_k^2 \delta(E - \hbar\Omega_k) \approx \frac{2\pi}{\hbar} \int_0^{k_{\max}} |F_k|^2 \rho_k dk \quad (10)$$

$$\Delta H_2 = \frac{2\pi}{\hbar} \frac{\alpha^2 \omega^2}{2\pi^2 \hbar c^3} \quad (11)$$

$$\begin{aligned} \Delta H_3 = & \frac{2\pi}{\hbar} \sum_k' \alpha^2 F_k^2 \left[\frac{\delta(E - \hbar\Omega_k)}{(\hbar\omega_0 + \hbar\Omega_k)(\hbar\Omega_k + \hbar\Omega_0)} \right. \\ & \left. + \frac{\delta(E - \hbar\omega_0)}{(\hbar\omega_0 + \hbar\Omega_k)(\hbar\Omega_0 + \hbar\omega_0)} \right] \approx \frac{2\pi}{\hbar} \frac{\alpha^2}{4(\hbar\omega_0)^2} \quad (12) \\ & \times \int_0^{k_{\max}} |F_k|^2 \rho_k dk + \frac{2\pi\alpha^2}{\hbar} \frac{\omega^2}{2\pi^2 \hbar c^3} \frac{|F_k|^2}{4(\hbar\omega_0)^2}. \end{aligned}$$

Here (10) is the result of Sparks et al. [2] for the two-magnon surface pit scattering. The next term (11) has been calculated by introducing a microwave slit width giving rise to a density of states for the photon of $\omega^2/2\hbar\pi^2 c^3$. This term can be interpreted to arise from Dicke's superradiance. The next term (12) arises because of the noncommutation of superradiance and the magnon-magnon surface scattering. For MnF_2 under normal circumstances [3], the (10) can vary from about 2 Gauss for a polished crystal to about 50 Gauss for a rough sample, (11) is about 15 Gauss and (12) is about $\lesssim 4$ Gauss.

Thus, we find that interesting new effects occur owing to the simultaneous presence of superradiance and the magnon scattering.

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